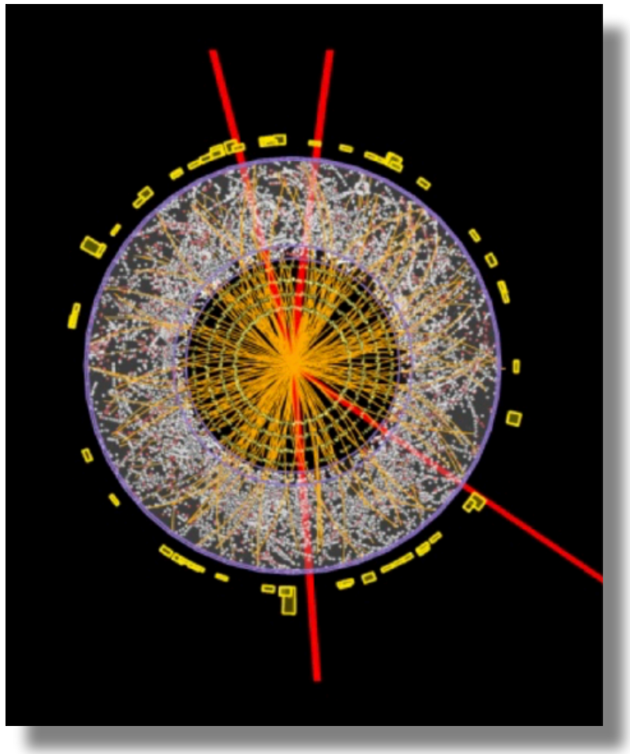
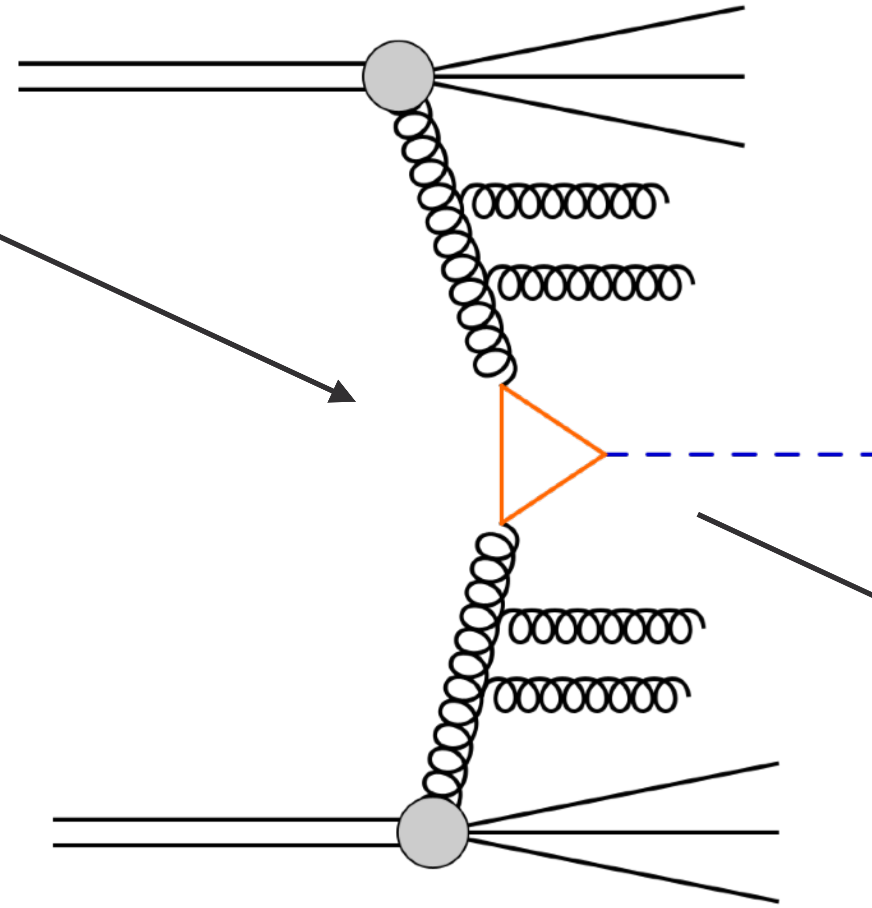
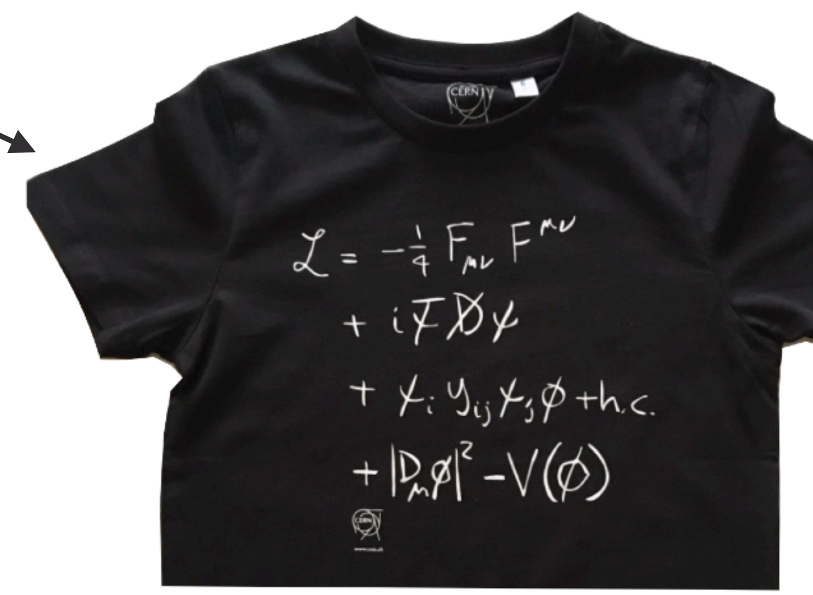
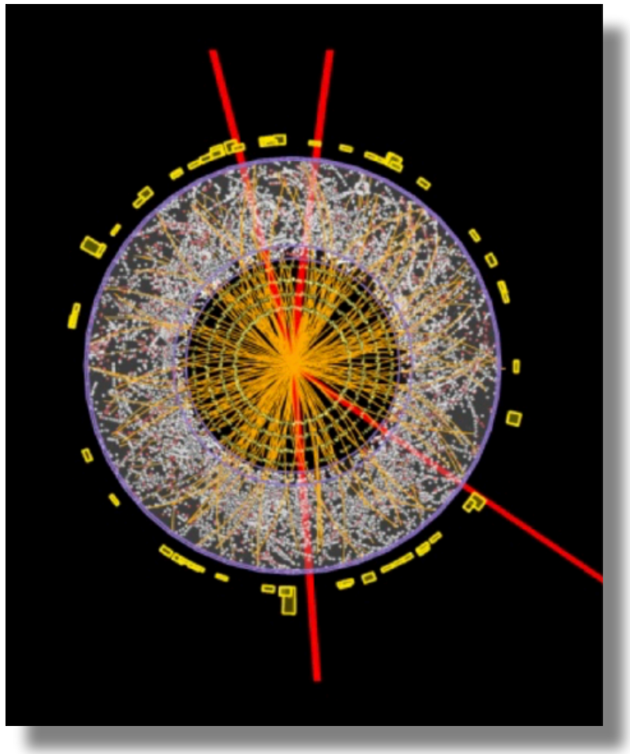


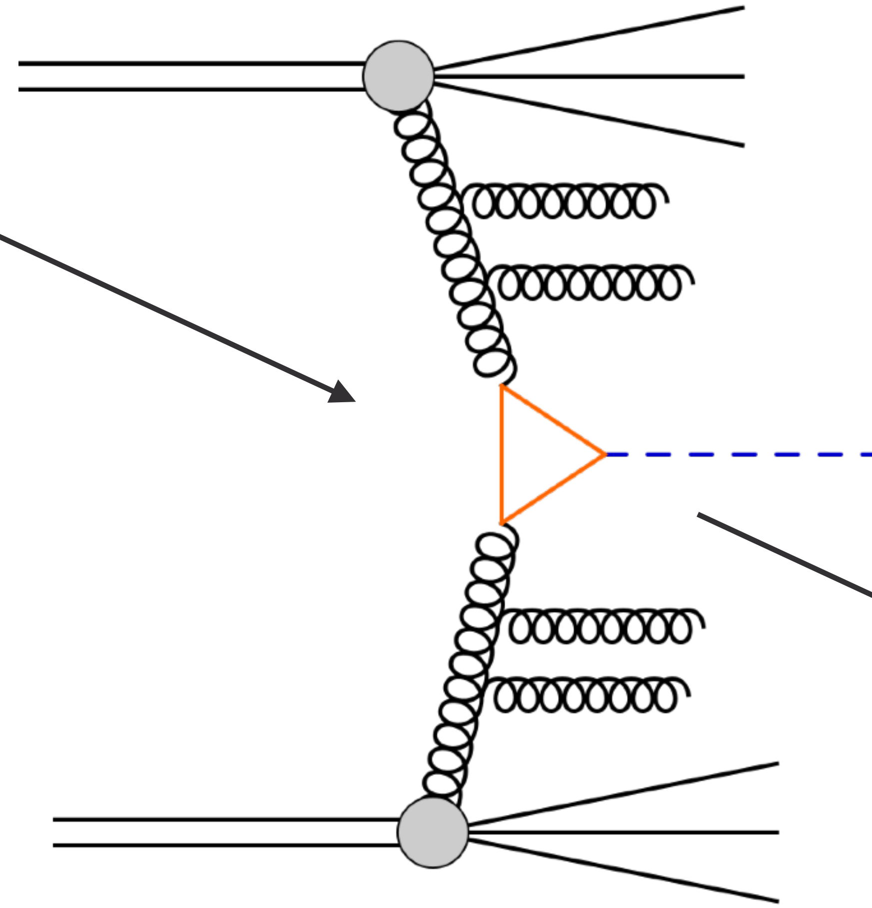


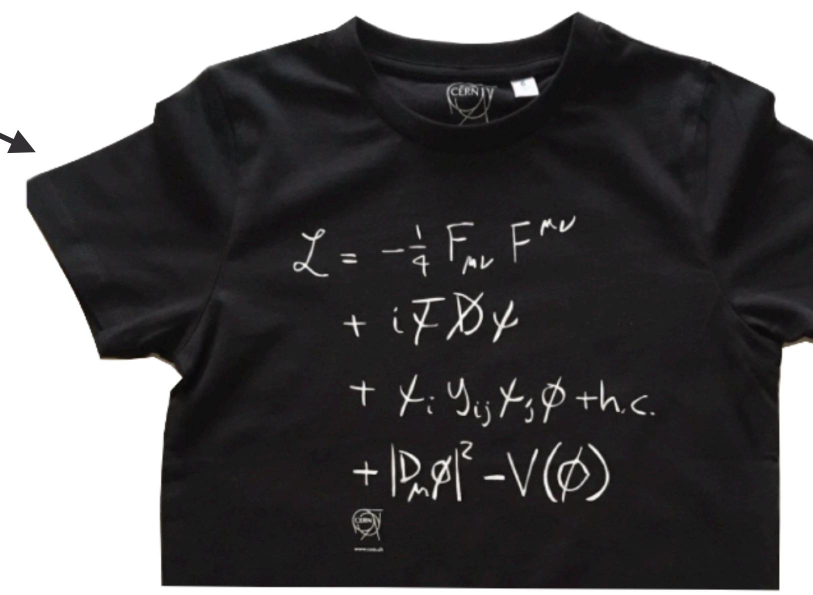


…*from collisions to the Higgs boson*

Fabrizio Caola
Rudolf Peierls Centre for Theoretical Physics & Wadham College

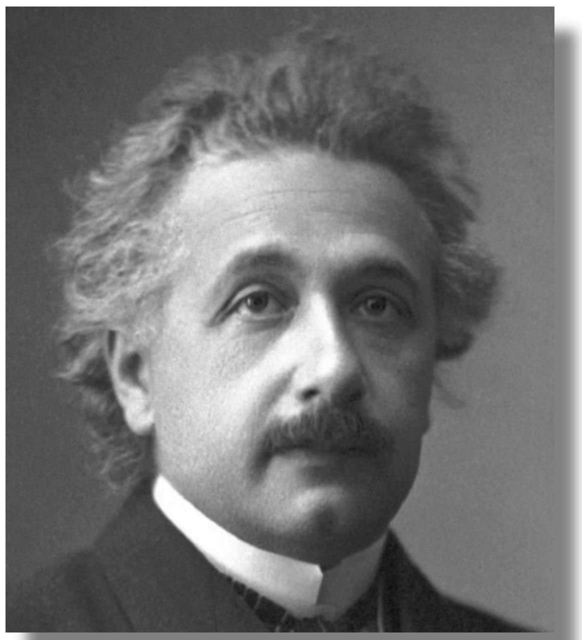






or: *how do quark and gluons interact and create a Higgs?*

The subatomic world at high energies



special relativity

+

quantum mechanics

= **QFT**

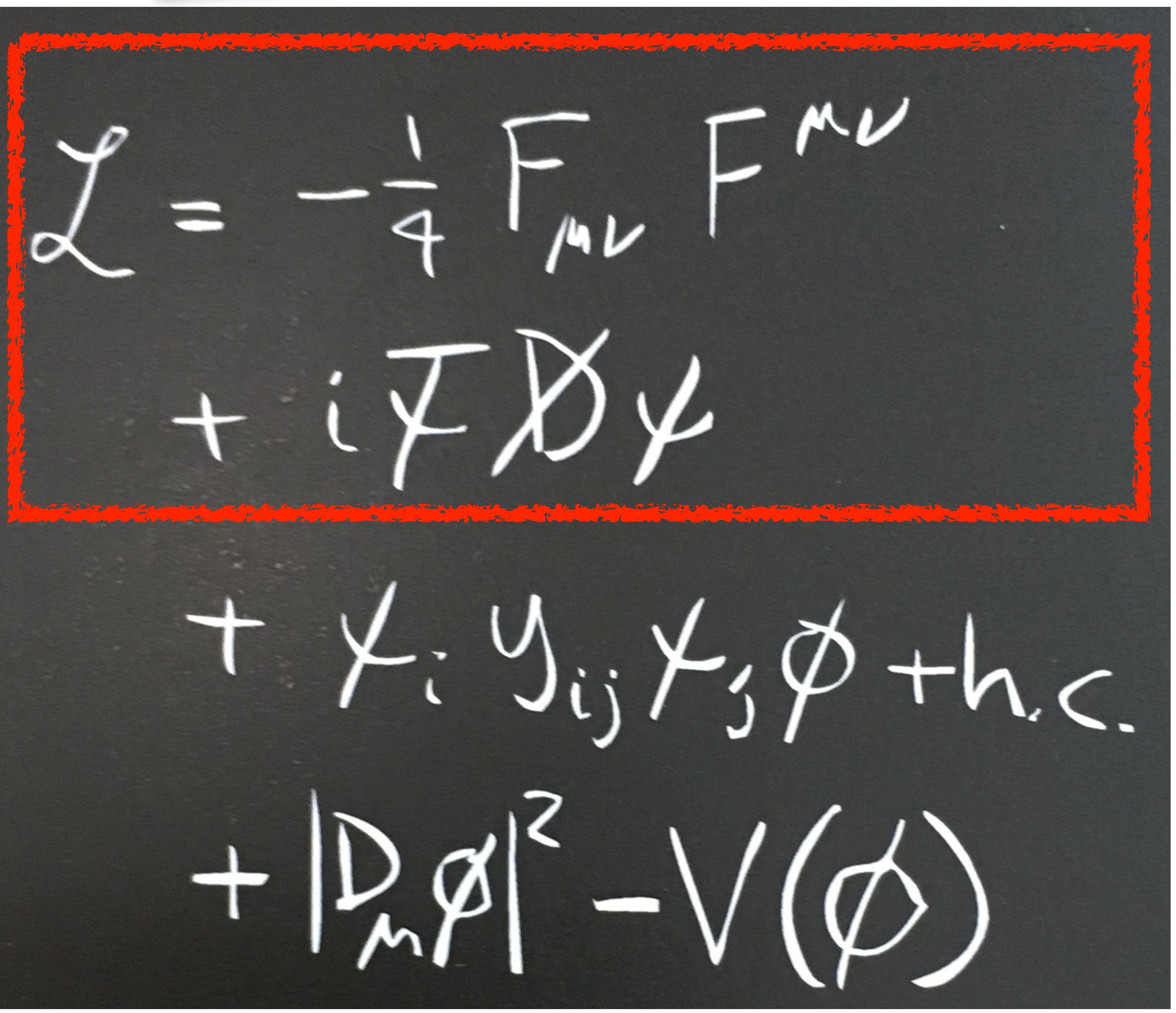
Quantum Field Theory

- Structure of fundamental interactions: very constrained
- Only freedom ~ particle content and symmetries

⟶ *first principles calculations*

Quantum chromodynamics

To study any process at the LHC:
we need to understand how quark and gluons interact



Quantum field theory for strong interactions → **quantum chromodynamics (QCD)**

A well-defined, well-established theory…

Quantum chromodynamics

To study any process at the LHC:
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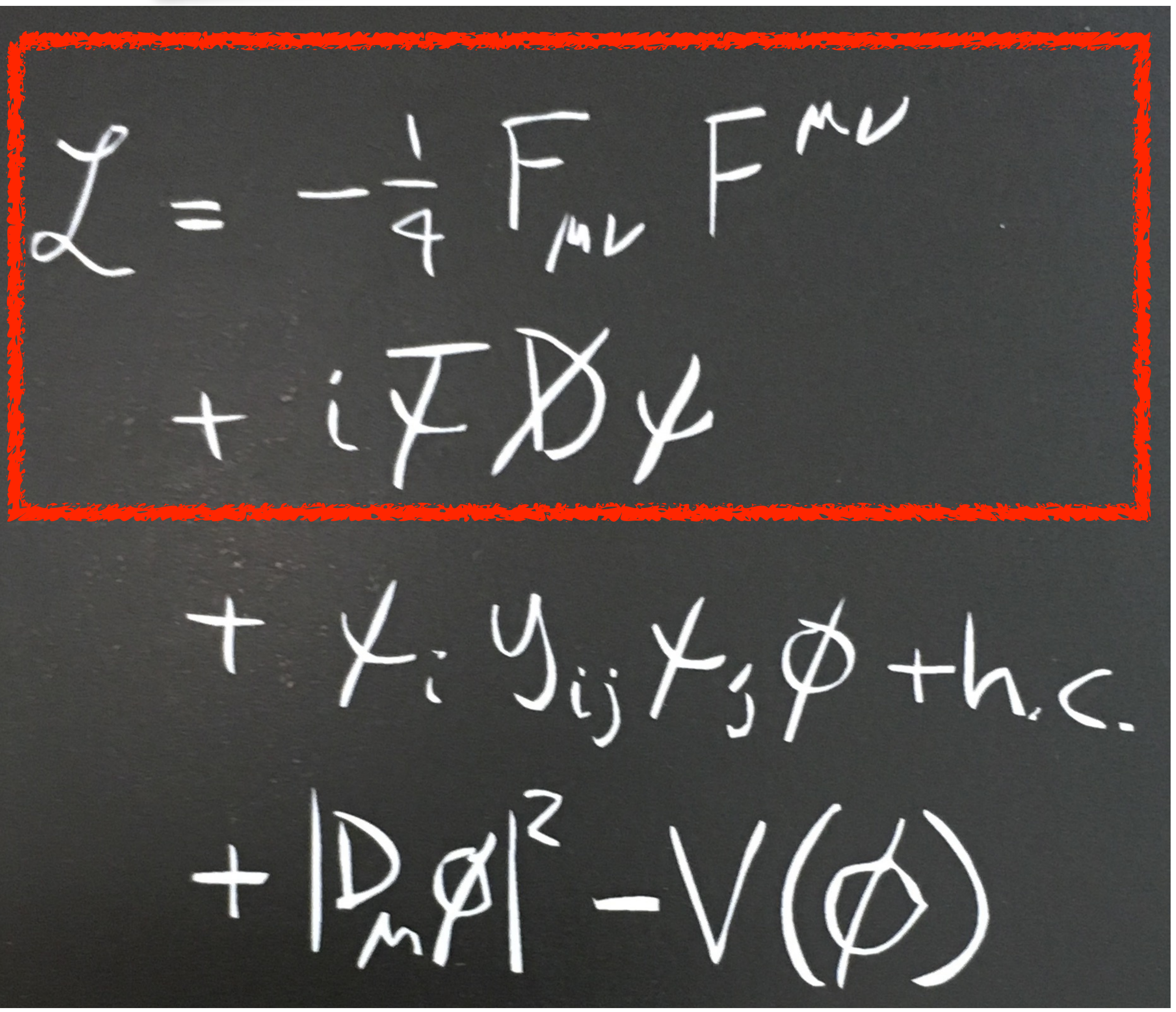
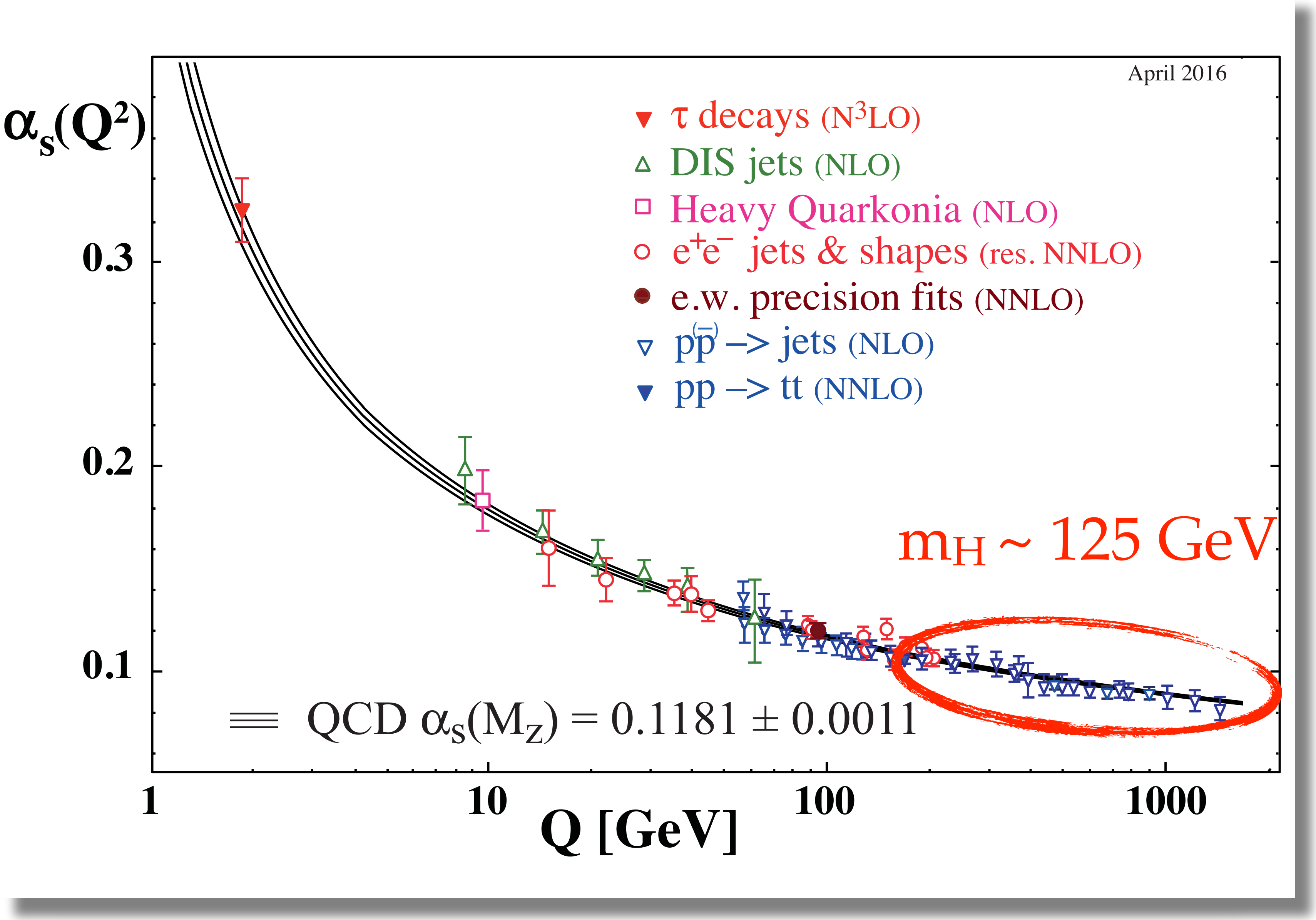


Quantum field theory for strong interactions → **quantum chromodynamics (QCD)**

A well-defined, well-established theory…
extremely hard to deal with

QCD for Higgs studies: weakly coupled

At high energies, QCD becomes *weakly coupled*



$$\alpha_s(m_P) \gg 1$$

$$\alpha_s(m_H) \sim 0.1$$

Study interactions as perturbation around theory of free quarks/gluons

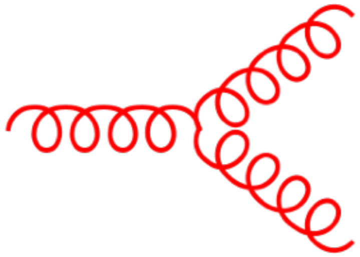
$\sigma = \sigma_0 + \alpha_s\,\sigma_1 + \alpha_s^2\,\sigma_2 + \ldots \rightarrow$ **perturbative QFT: *Feynman diagrams***

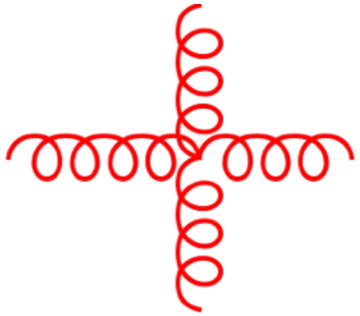
Perturbation theory and Feynman diagrams

$$I \rightarrow F \qquad \sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \ldots$$

- For a given QFT $\rightarrow$ set of basic building blocks

QCD, only gluons





Perturbation theory and Feynman diagrams

$I \rightarrow F$ $\qquad \sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \ldots$

- For a given QFT → set of basic building blocks

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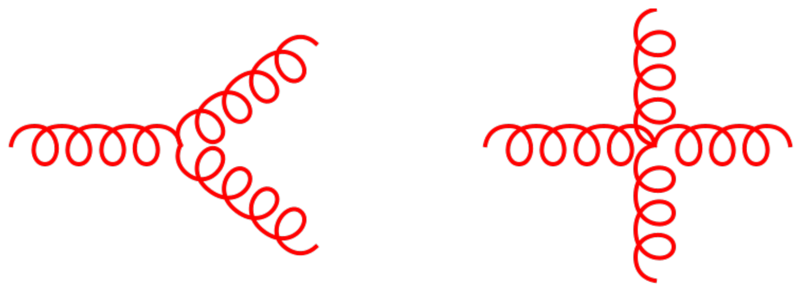
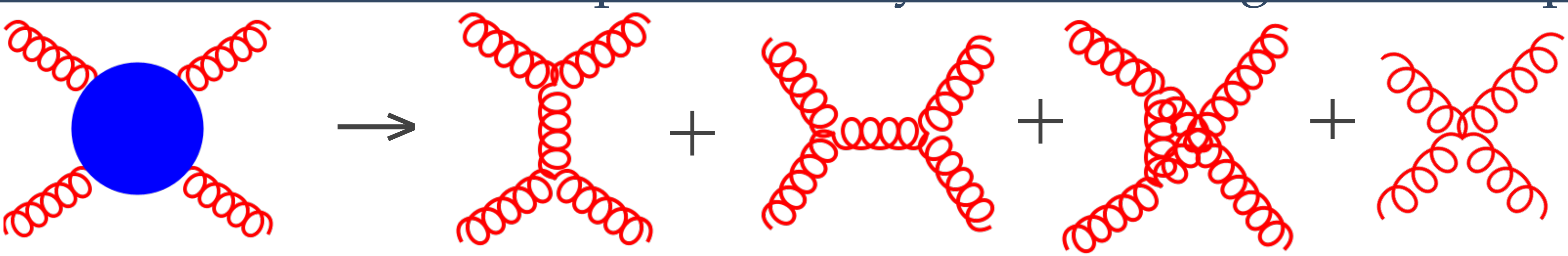


- σ_0: connect *I* to *F*, in all possible ways, minimising closed loops



Perturbation theory and Feynman diagrams

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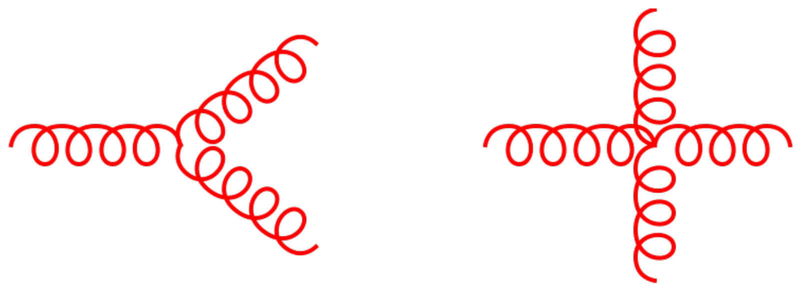

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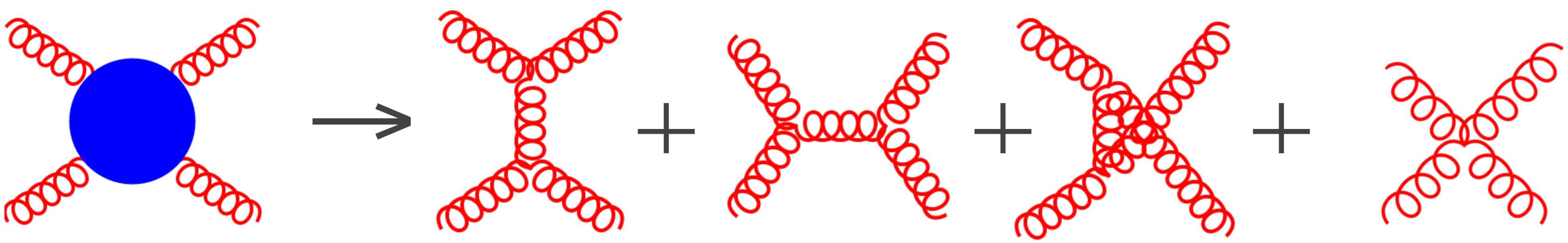
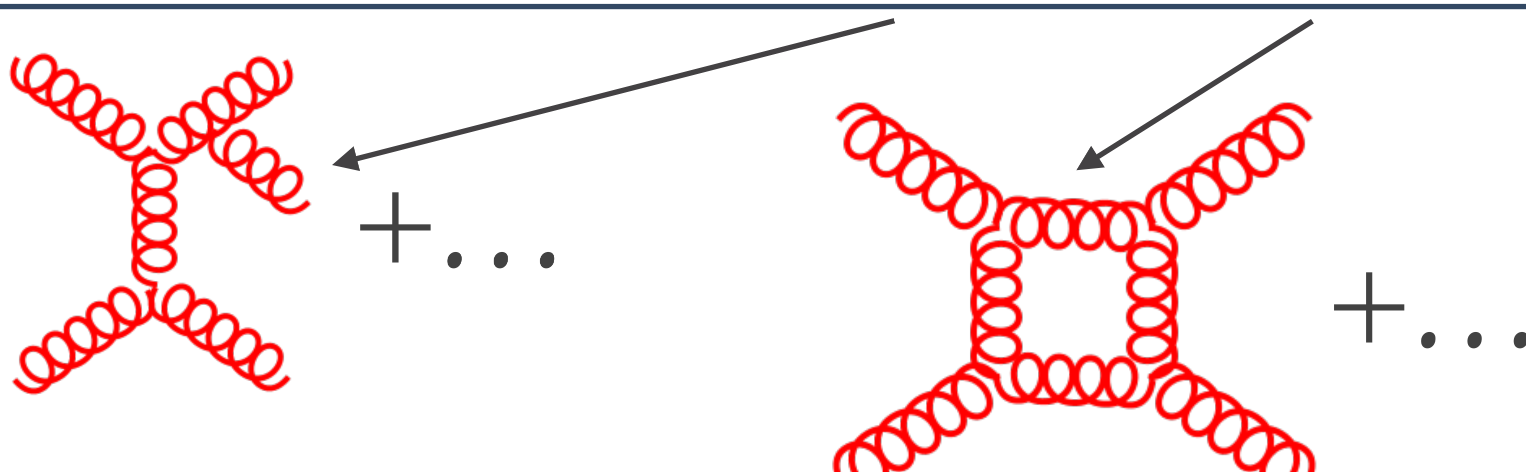


- Higher orders: dress with real and virtual quark and gluons

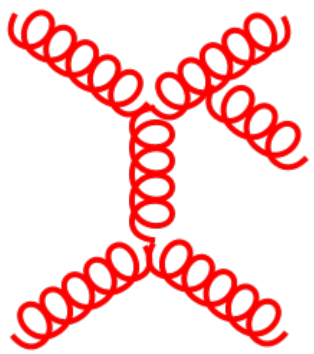


one extra leg/loop per order

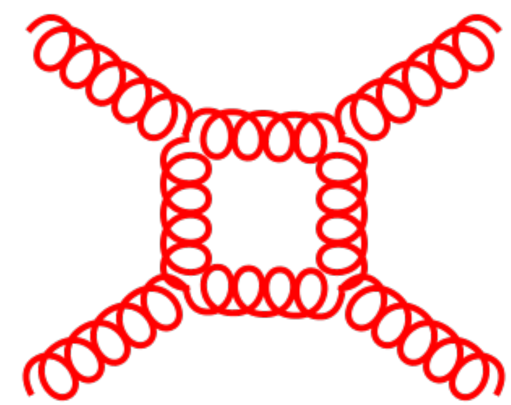
Perturbation theory and Feynman diagrams

$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s{}^2 \sigma_2 + \ldots$$

- *Feynman rules*: associate to each diagram an analytic formula



``Tree" diagrams: simple rational functions of momenta / polarisations



``Loop": must integrate over momenta of particles in the loop → non trivial transcendental functions

- Very well-understood procedure since the `60s
- Fully algorithmic

The punch-line:

To compute any precise theoretical prediction for any LHC process →
need to compute Feynman diagrams with many legs/loops

How many?

- We want to test Higgs interactions at the few percent…

$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \ldots,\ \alpha_s \sim 0.1$

Leading Order (LO) → very imprecise

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Next-to-Leading Order (NLO) → ~ 10%

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Leading Order (LO) → very imprecise

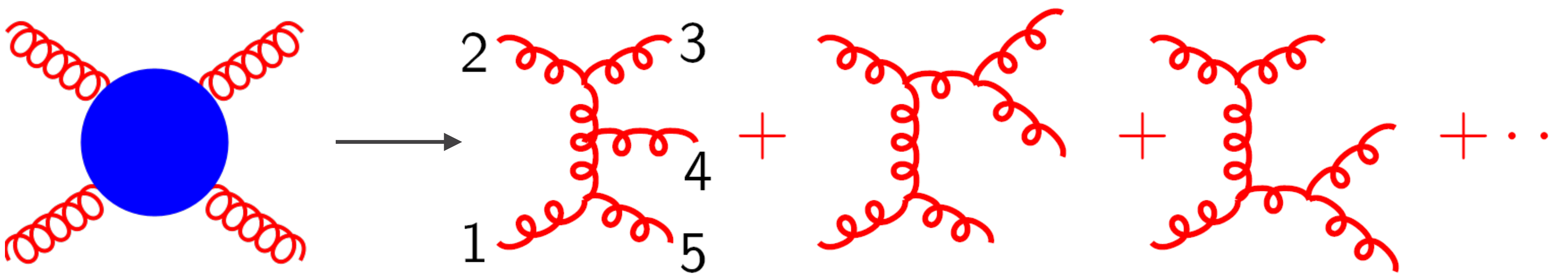
Next-to-Leading Order (NLO) → ~ 10%

Need Next-to-next-to-leading order (NNLO) and beyond for precision

…in principle: *compute a bunch of diagrams with extra legs/loops*

… in practice: back to our example

Dress with one real gluon



+ 22 similar terms

```
amp =

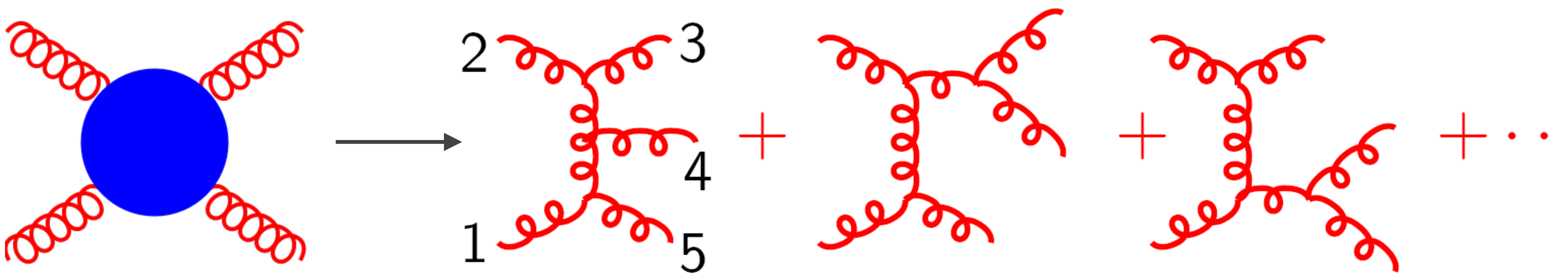
    + F(CiE1,Ci2,CiE4)*F(CiE2,Ci2,CiXX)*F(CiE3,CiE5,CiXX) * (
       - e1.e3*e2.e5*e4.p1*s14^-1
       - 1/2*e1.e3*e2.e5*e4.p4*s14^-1
       - 1/2*e1.e4*e2.e3*e5.p1*s14^-1
       + 1/2*e1.e4*e2.e3*e5.p4*s14^-1
       + 1/2*e1.e4*e2.e5*e3.p1*s14^-1
       - 1/2*e1.e4*e2.e5*e3.p4*s14^-1
       + e1.e5*e2.e3*e4.p1*s14^-1
       + 1/2*e1.e5*e2.e3*e4.p4*s14^-1
       - 1/2*e1.p1*e2.e3*e4.e5*s14^-1
       + 1/2*e1.p1*e2.e5*e3.e4*s14^-1
       - e1.p4*e2.e3*e4.e5*s14^-1
       + e1.p4*e2.e5*e3.e4*s14^-1
       )

    + F(CiE1,Ci2,CiE4)*F(CiE2,Ci2,Ci4)*F(CiE3,CiE5,Ci4) * (
       - 2*e1.e2*e3.e5*e4.p1*p1.p3*s14^-1*s124^-1
       - 2*e1.e2*e3.e5*e4.p1*p1.p4*s14^-1*s124^-1
       + 2*e1.e2*e3.e5*e4.p1*p2.p3*s14^-1*s124^-1
       - 2*e1.e2*e3.e5*e4.p1*p3.p4*s14^-1*s124^-1
       - e1.e2*e3.e5*e4.p4*p1.p3*s14^-1*s124^-1
       - e1.e2*e3.e5*e4.p4*p1.p4*s14^-1*s124^-1
       + e1.e2*e3.e5*e4.p4*p2.p3*s14^-1*s124^-1
       - e1.e2*e3.e5*e4.p4*p3.p4*s14^-1*s124^-1
       + e1.e2*e3.p1*e4.p1*e5.p1*s14^-1*s124^-1
       - 3*e1.e2*e3.p1*e4.p1*e5.p2*s14^-1*s124^-1
       + e1.e2*e3.p1*e4.p1*e5.p3*s14^-1*s124^-1
       + e1.e2*e3.p1*e4.p1*e5.p4*s14^-1*s124^-1
       + 1/2*e1.e2*e3.p1*e4.p4*e5.p1*s14^-1*s124^-1
       - 3/2*e1.e2*e3.p1*e4.p4*e5.p2*s14^-1*s124^-1
       + 1/2*e1.e2*e3.p1*e4.p4*e5.p3*s14^-1*s124^-1
       + 1/2*e1.e2*e3.p1*e4.p4*e5.p4*s14^-1*s124^-1
       + 3*e1.e2*e3.p2*e4.p1*e5.p1*s14^-1*s124^-1
```

$\longrightarrow$ *Rational function of momenta and polarisations*

… in practice: back to our example

Dress with one real gluon



+ 22 similar terms

```
- 1/2*e1.e3*e2.e4*e5.p4*s134^-1
- 1/2*e1.e3*e2.e5*e4.p1*s134^-1
- e1.e3*e2.e5*e4.p2*s134^-1
- 1/2*e1.e3*e2.e5*e4.p3*s134^-1
- 1/2*e1.e3*e2.e5*e4.p4*s134^-1
+ e1.e3*e2.p1*e4.e5*s134^-1
+ 1/2*e1.e3*e2.p2*e4.e5*s134^-1
+ e1.e3*e2.p3*e4.e5*s134^-1
+ e1.e3*e2.p4*e4.e5*s134^-1
- e1.e5*e2.p1*e3.e4*s134^-1
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- e1.e5*e2.p3*e3.e4*s134^-1
- e1.e5*e2.p4*e3.e4*s134^-1
+ 1/2*e1.p1*e2.e5*e3.e4*s134^-1
+ e1.p2*e2.e5*e3.e4*s134^-1
+ 1/2*e1.p3*e2.e5*e3.e4*s134^-1
+ 1/2*e1.p4*e2.e5*e3.e4*s134^-1
)

+ F(CiE1,CiXX,CiE4)*F(CiE3,CiE2,Ci2)*F(CiE5,Ci2,CiXX) * (
   + e1.e2*e3.p2*e4.e5*s23^-1
   + 1/2*e1.e2*e3.p3*e4.e5*s23^-1
   - 1/2*e1.e3*e2.p2*e4.e5*s23^-1
   - e1.e3*e2.p3*e4.e5*s23^-1
   + 1/2*e1.e5*e2.e3*e4.p2*s23^-1
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   - e1.e5*e2.e4*e3.p2*s23^-1
   - 1/2*e1.e5*e2.e4*e3.p3*s23^-1
   + 1/2*e1.e5*e2.p2*e3.e4*s23^-1
   + e1.e5*e2.p3*e3.e4*s23^-1
   - 1/2*e1.p2*e2.e3*e4.e5*s23^-1
   + 1/2*e1.p3*e2.e3*e4.e5*s23^-1
   );
```

98 pages analytic formula!

More legs…

Final state gluons	2	3	4	5	6	…
# diagrams	4	25	220	2485	34300	…

Explosion of terms

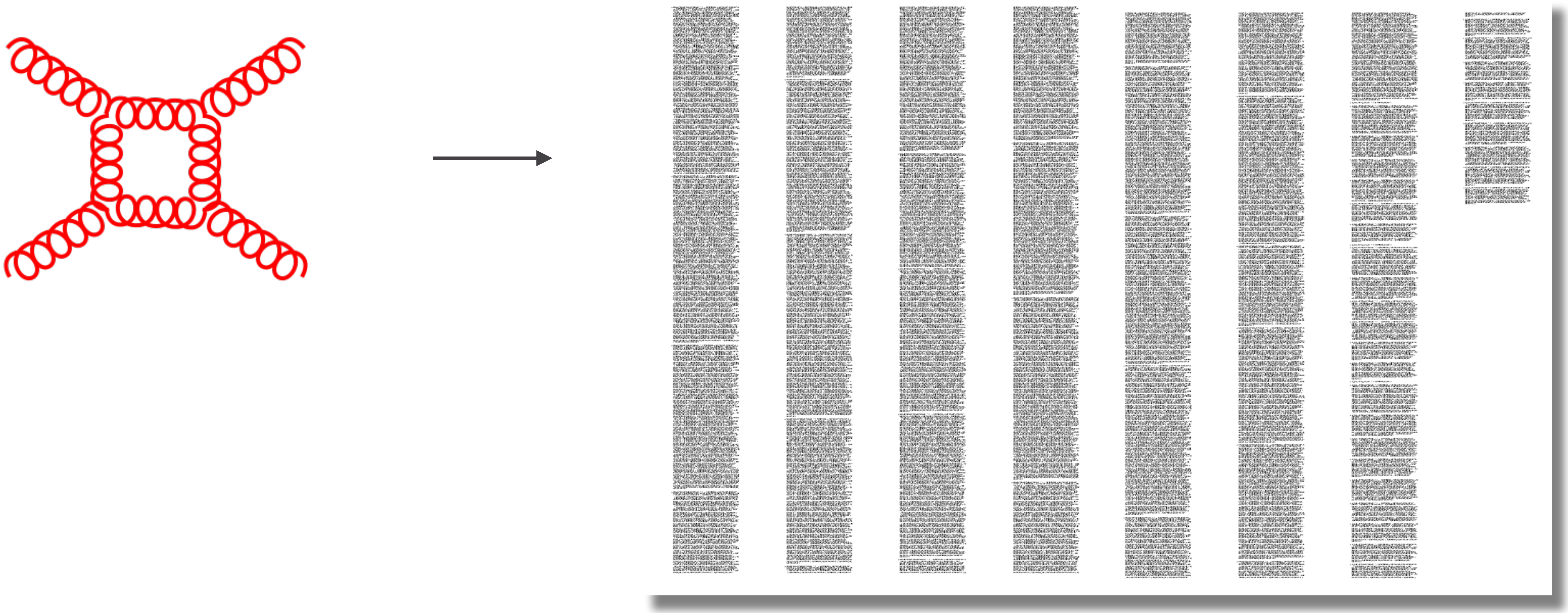
Supercollider physics (1984) 617

It is apparent that these questions are amenable to detailed investigation with the aid of realistic Monte Carlo simulations. Given the elementary two→three cross sections and reasonable parametrizations of the fragmentation functions, this exercise can be carried out with some degree of confidence.

For multijet events containing more than three jets, the theoretical situation is considerably more primitive. A specific question of interest concerns the QCD four-jet background to the detection of W^+W^- pairs in their nonleptonic decays. **The cross sections for the elementary two→four processes have not been calculated, and their complexity is such that they may not be evaluated in the foreseeable future.** It is worthwhile to seek estimates of the four-jet cross sections, even if these are only reliable in restricted regions of phase space.

Prospect for theoretical calculations 30y ago

Loops?



- Very difficult integrals…
- … when you compute them you get infinity!

<u>in principle</u>, last century physics tells us what to do…

… <u>in practice</u>, we don't go very far

Typical expectation: hopeless

My first interaction with a Nobel Prize winner…

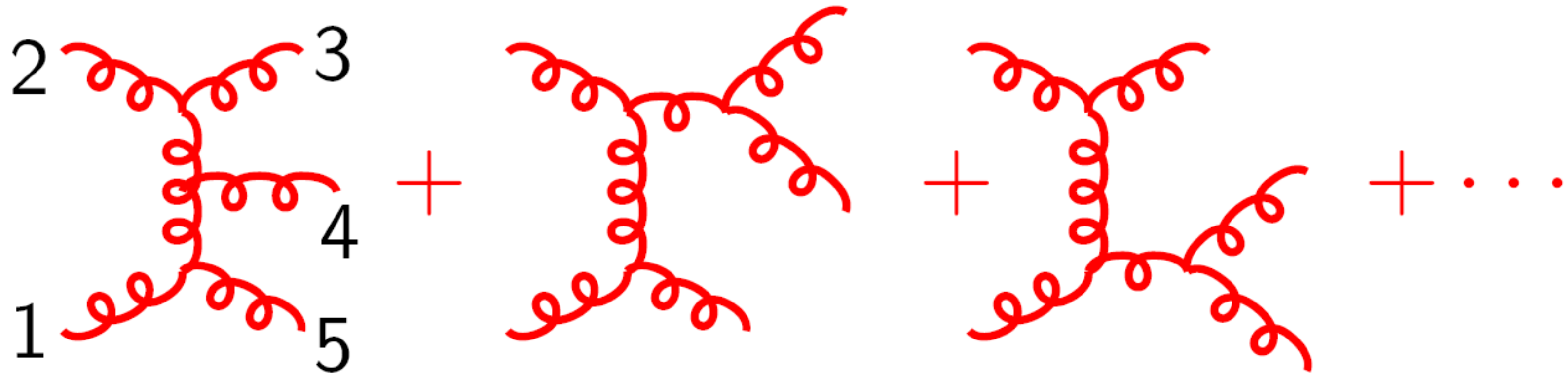
- Him, to friend A: *what are you working on?*
- My friend: *X and Y […]*
- Him: *Fascinating […] Keep on the good work!*

Similar pattern with friend B, C… until it is my turn

- Him: *and you, what are you doing?*
- Me: I am trying to do precision physics at the LHC
- Him, genuinely worried for me: *my dear boy, no! You should change topic, the LHC is a messy environment, we are going to discover stuff but we cannot do precision physics there!*

if we kept relying on `70s understanding of QFT, he had a point…

… but in the meantime



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       + 1/2*e1.e4*e2.e3*e5.p4*s14^-1
       + 1/2*e1.e4*e2.e5*e3.p1*s14^-1
       - 1/2*e1.e4*e2.e5*e3.p4*s14^-1
       + e1.e5*e2.e3*e4.p1*s14^-1
       + 1/2*e1.e5*e2.e3*e4.p4*s14^-1
       - 1/2*e1.p1*e2.e3*e4.e5*s14^-1
       + 1/2*e1.p1*e2.e5*e3.e4*s14^-1
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       - 2*e1.e2*e3.e5*e4.p1*p1.p4*s14^-1*s124^-1
       + 2*e1.e2*e3.e5*e4.p1*p2.p3*s14^-1*s124^-1
       - 2*e1.e2*e3.e5*e4.p1*p3.p4*s14^-1*s124^-1
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       + e1.e2*e3.p1*e4.p1*e5.p3*s14^-1*s124^-1
       + e1.e2*e3.p1*e4.p1*e5.p4*s14^-1*s124^-1
       + 1/2*e1.e2*e3.p1*e4.p4*e5.p1*s14^-1*s124^-1
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       + 1/2*e1.e2*e3.p1*e4.p4*e5.p3*s14^-1*s124^-1
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       + 3*e1.e2*e3.p2*e4.p1*e5.p1*s14^-1*s124^-1
```

=

Massive simplification!

$$A_5^{\text{tree}}(1^{\pm},2^{+},3^{+},4^{+},5^{+}) = 0$$

$$A_5^{\text{tree}}(1^{-},2^{-},3^{+},4^{+},5^{+}) = i\frac{\langle 12\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

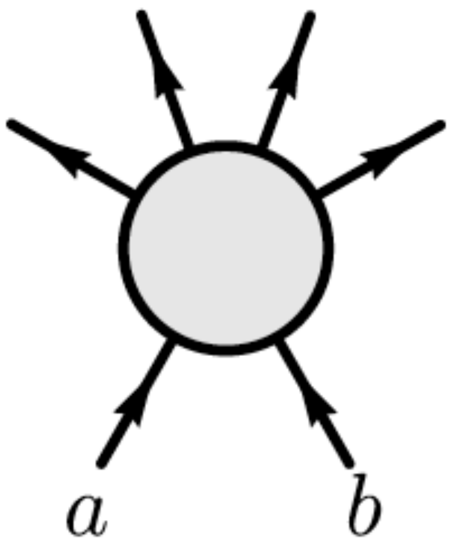
$$A_5^{\text{tree}}(1^{-},2^{+},3^{-},4^{+},5^{+}) = i\frac{\langle 13\rangle^4}{\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle}$$

Same physical content

Massive simplifications in tree amplitudes

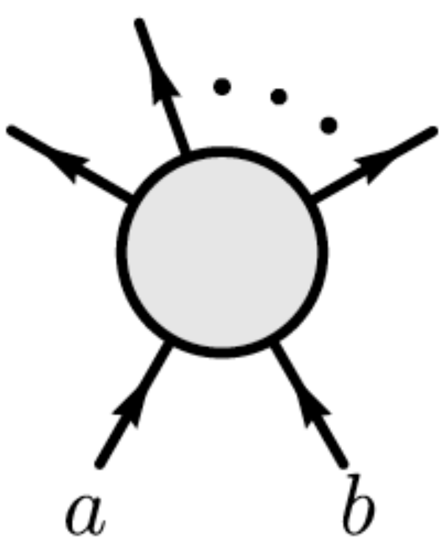
Similar simplification persist at higher multiplicity

Simplest helicity configuration:



$$= \frac{\langle a\, b\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\langle 3\, 4\rangle\langle 4\, 5\rangle\langle 5\, 6\rangle\langle 6\, 1\rangle}$$

Sum of 220 Feynman diagrams!



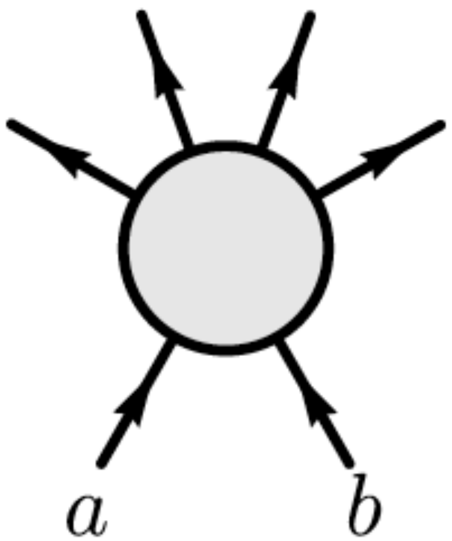
$$= \frac{\langle a\, b\rangle^4}{\langle 1\, 2\rangle\langle 2\, 3\rangle\langle 3\, 4\rangle\langle 4\, 5\rangle \cdots \langle n\, 1\rangle}$$

Sum of ~n! diagrams

Massive simplifications in tree amplitudes

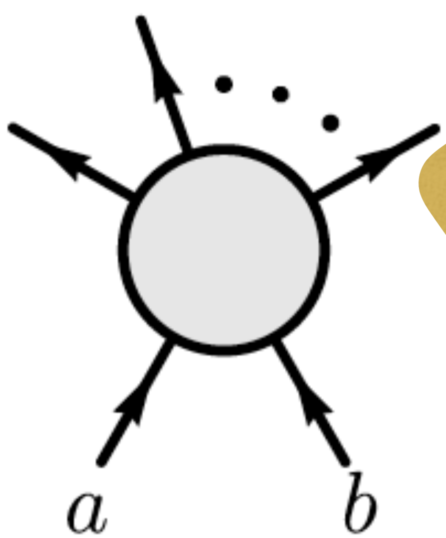
Similar simplification persist at higher multiplicity

Simplest helicity configuration:



$$= \frac{\langle a\,b\rangle^4}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,6\rangle \ldots}$$

220 Feynman diagrams!



$$\frac{\langle a\,b\rangle^4}{\ldots\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle \cdots \langle n\,1\rangle}$$

Sum of ~n! diagrams

What is going on?

QFT in the XXI century

Structure of QFT extremely rigid → very much constrained by special relativity and quantum mechanics

- Special relativity: *everything is local* → tree-level results can only have a well-defined set of simple singularities
- Quantum mechanics: *unitarity* → what happens at singular points entirely determined by trees with lower multiplicity

Completely hidden in Feynman diagrams!

↓

Trees have a natural recursive nature, that can be fully exploited to reconstruct the result for *n+1* legs from the *n-leg* one.

QFT in the XXI century

Structure of QFT extremely rigid → very much constrained by special relativity and quantum mechanics

- <u>Special relativity</u>: *everything is local* → tree-level
have a well-defined set of simple singula

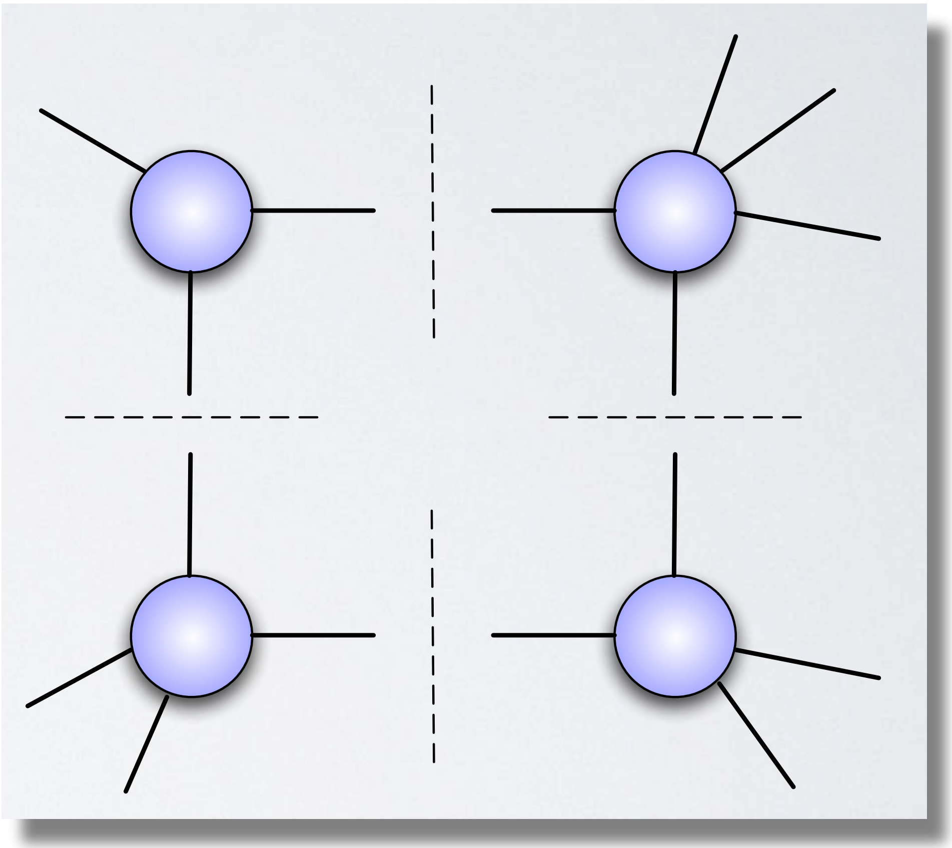
- <u>Quantum mechanics</u>: *unitarity* singular points
entirely determined by t ultiplicity

Completely hidde grams!

Trees: problem solved

Trees have a natural recursive nature, that can be fully exploited to reconstruct the result for *n+1* legs from the *n-leg* one. *Any process*.

… what about loops?

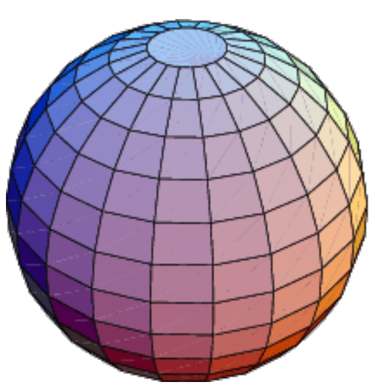


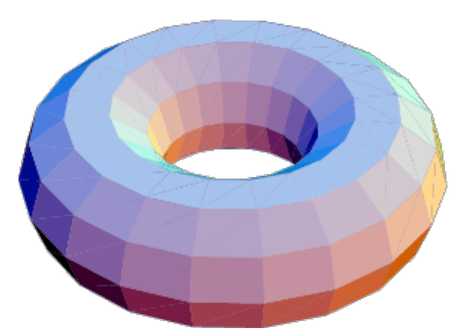
Similar ideas:

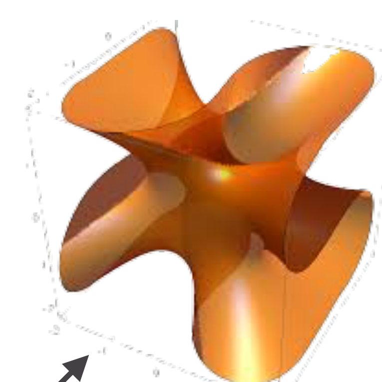
- ``cut open a loop'' → tree
- use smart trees → simplify dramatically the function to integrate

Integrals?

- one loop → solved long time ago
- higher loop → very complicated, but they seem to have nice geometrical structures… *can we understand this?*





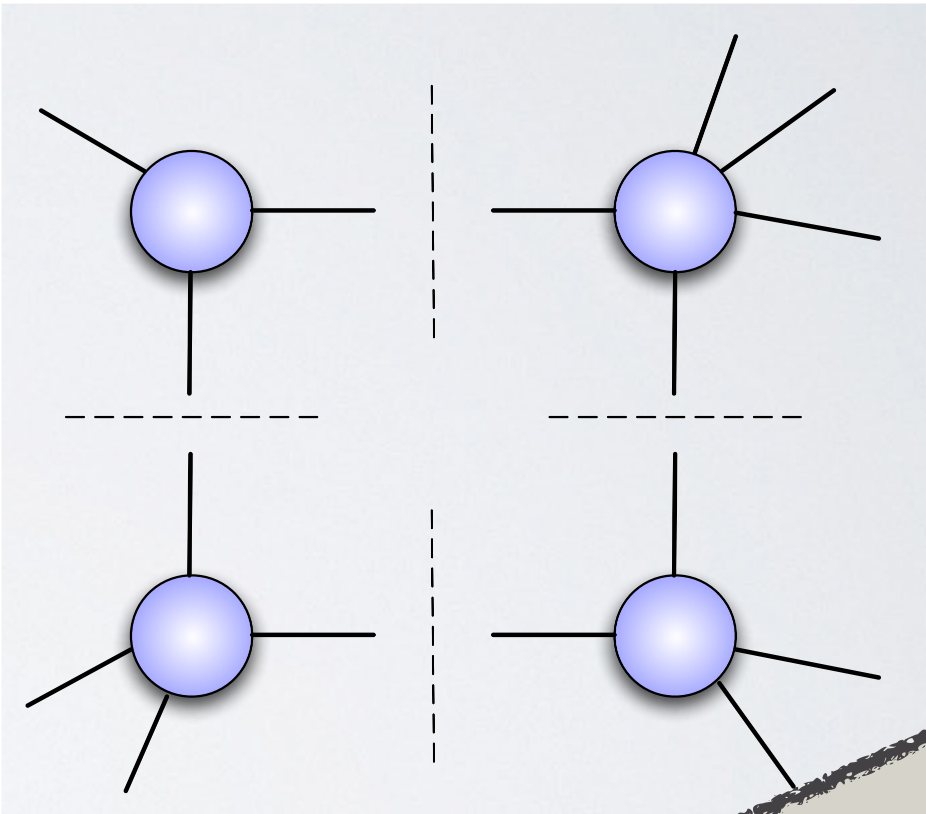


?

One-loop

Multi-loop

… what about loops?



Similar ideas:

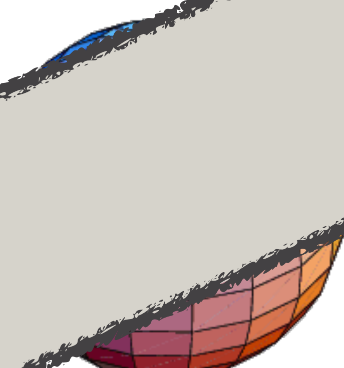
- ``cut open a loop'' → tree
- use smart trees → si… dramatically th… …egrate

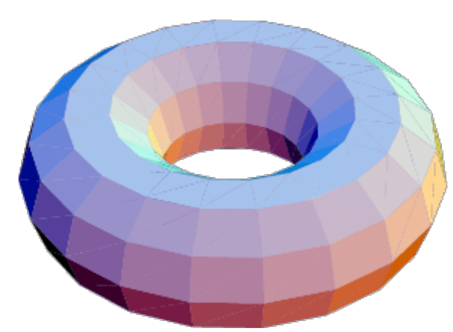
Integr…

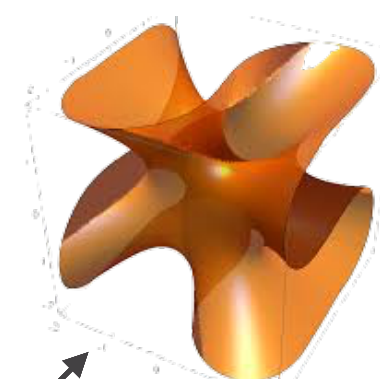
…lved long time ago

…loop → very complicated, but …ey seem to have nice geometrical structures… *can we understand this?*

A lot of progress







?

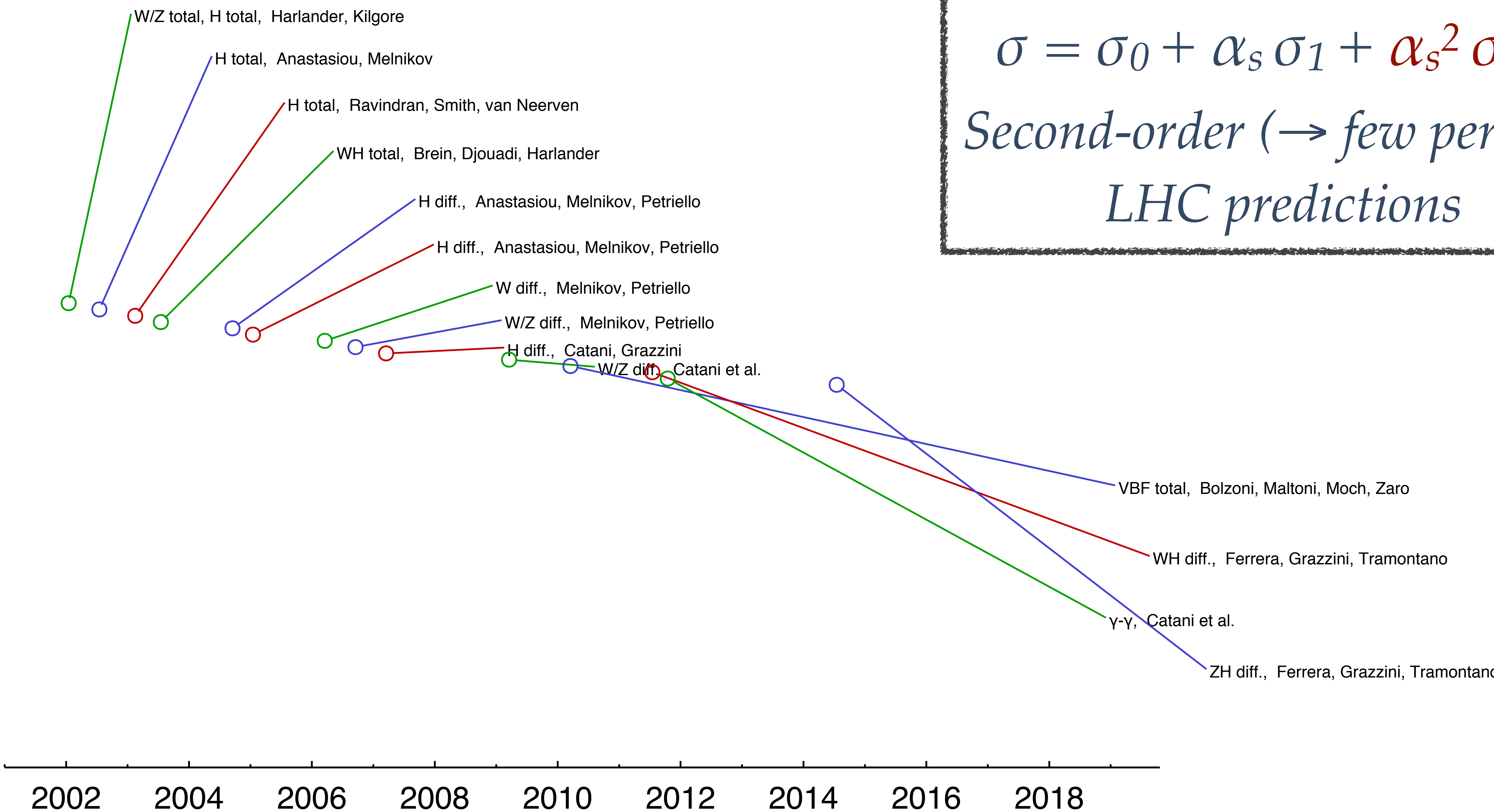
One-loop

Multi-loop

Precision physics at the LHC: timeline

$$\sigma = \sigma_0 + \alpha_s \sigma_1 + \alpha_s^2 \sigma_2 + \ldots$$

Second-order (→ few percent) LHC predictions



[thanks to Gavin for the graphics]

Precision physics at the LHC: timeline

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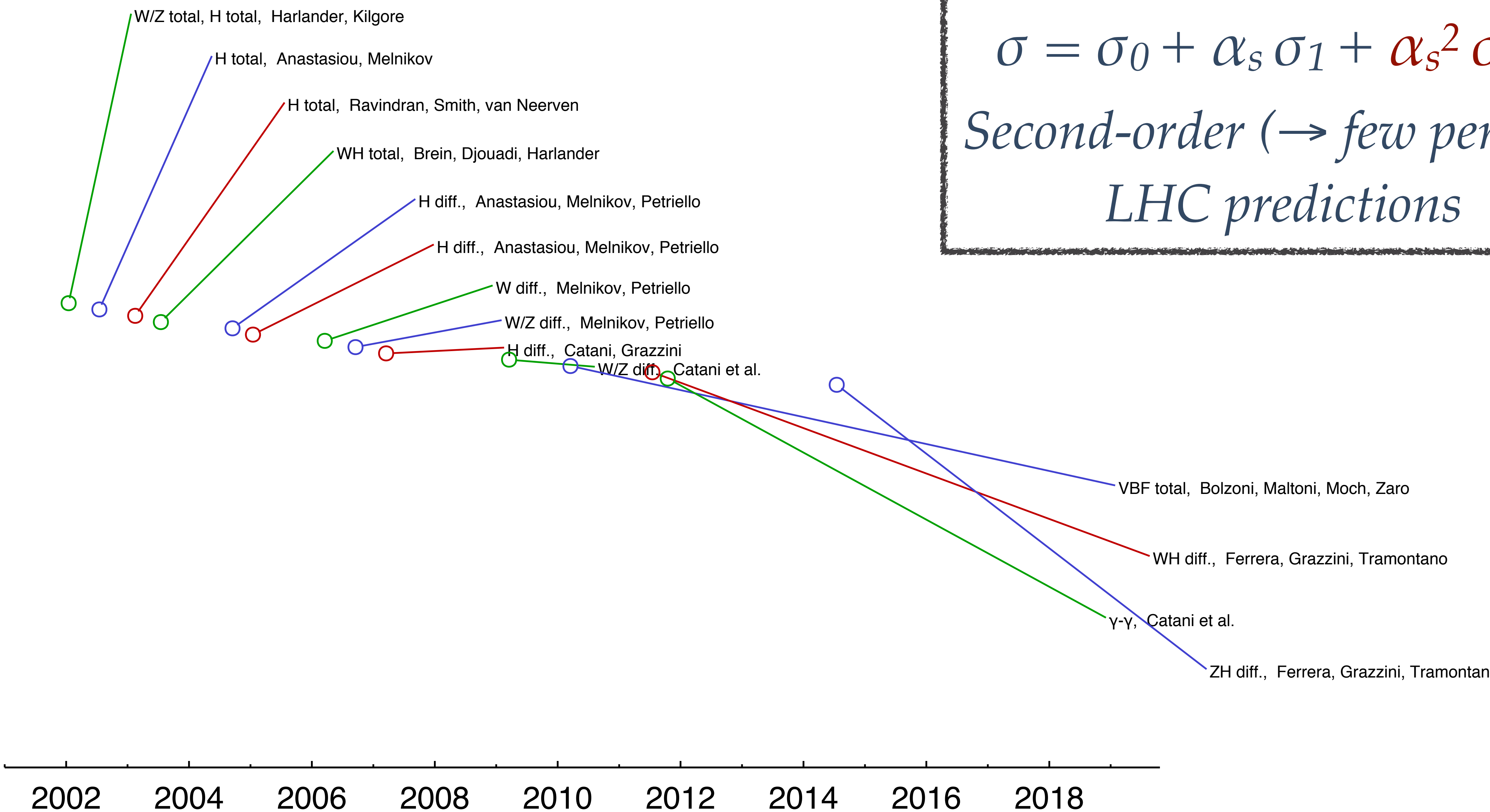
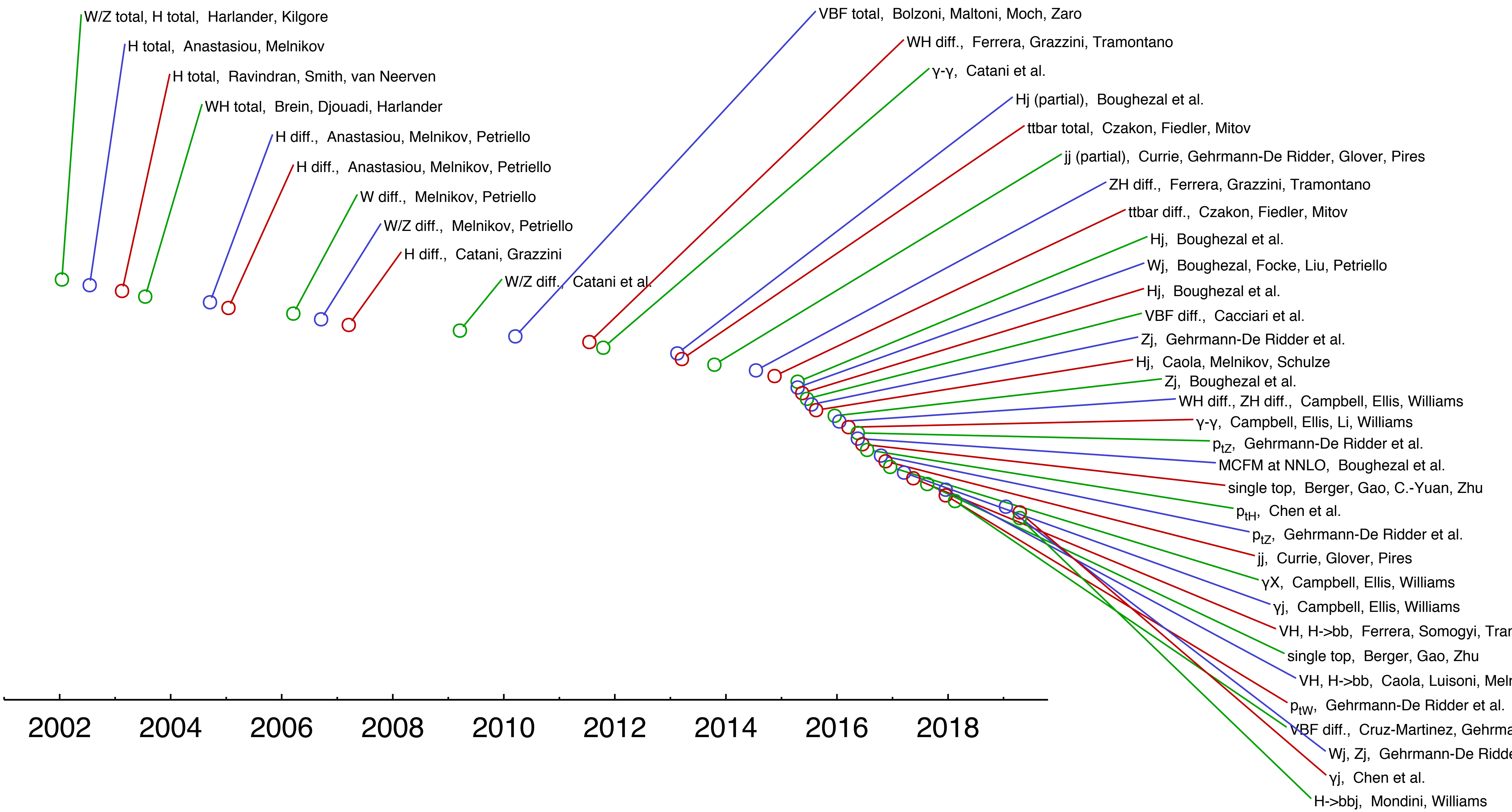
[*thanks to Gavin for the graphics*]



First steps towards dealing with infinities in generic processes

New ideas for 2-loop

Precision physics at the LHC: timeline



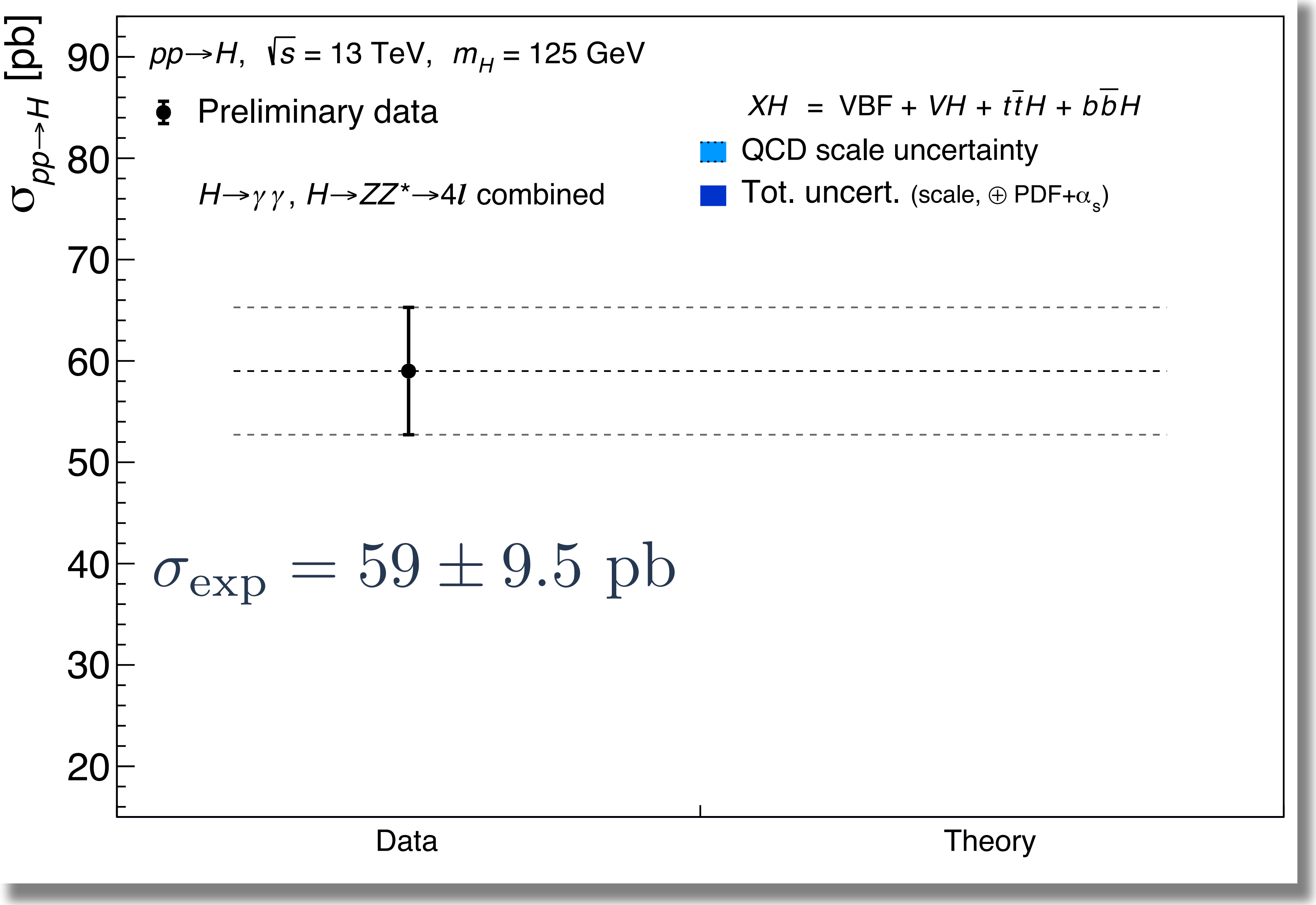
[thanks to Gavin for the graphics]

Explosion of higher order results

precision physics at the LHC is possible

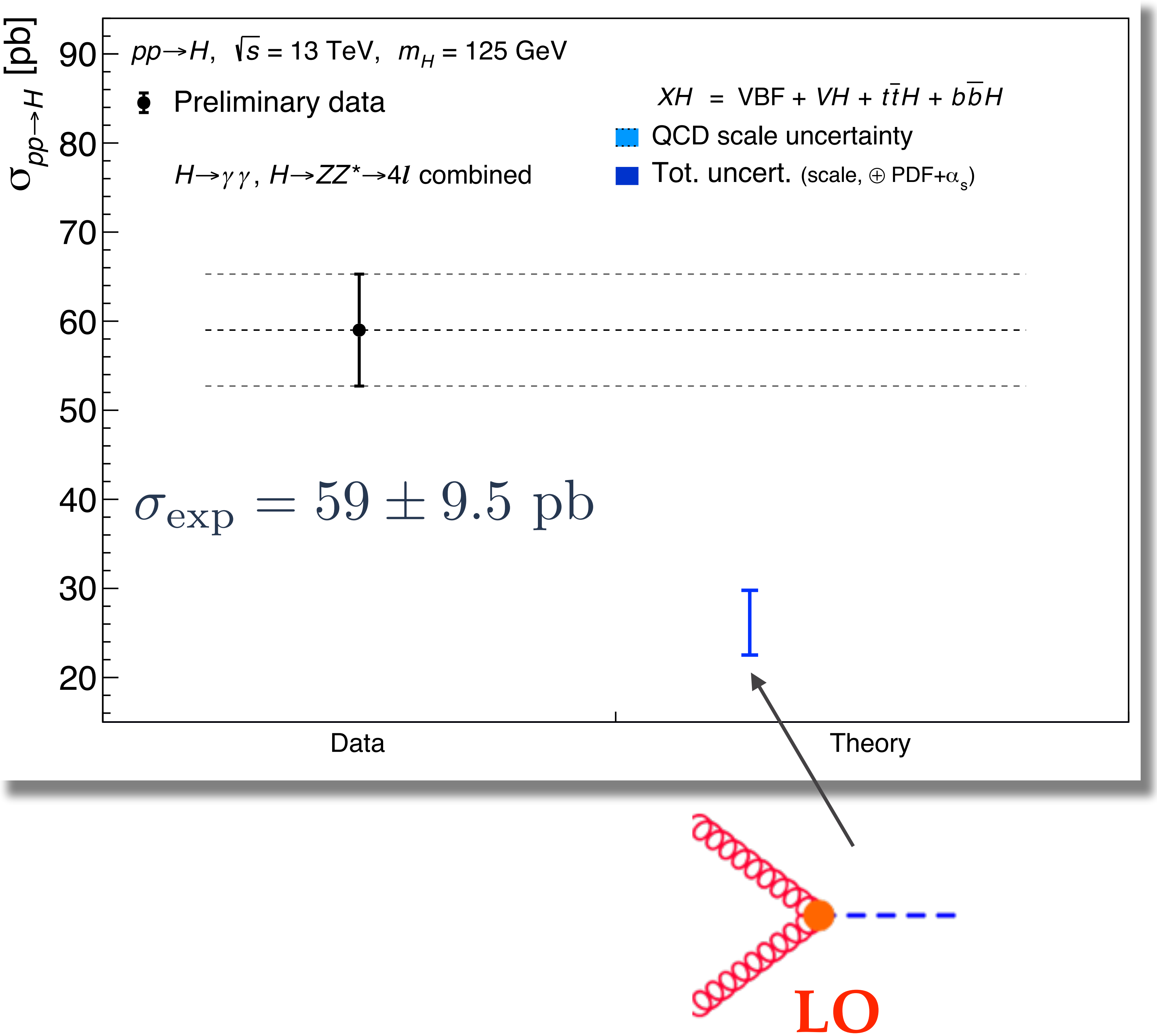
Precision and the Higgs

Higgs boson production: today



Precision and the Higgs

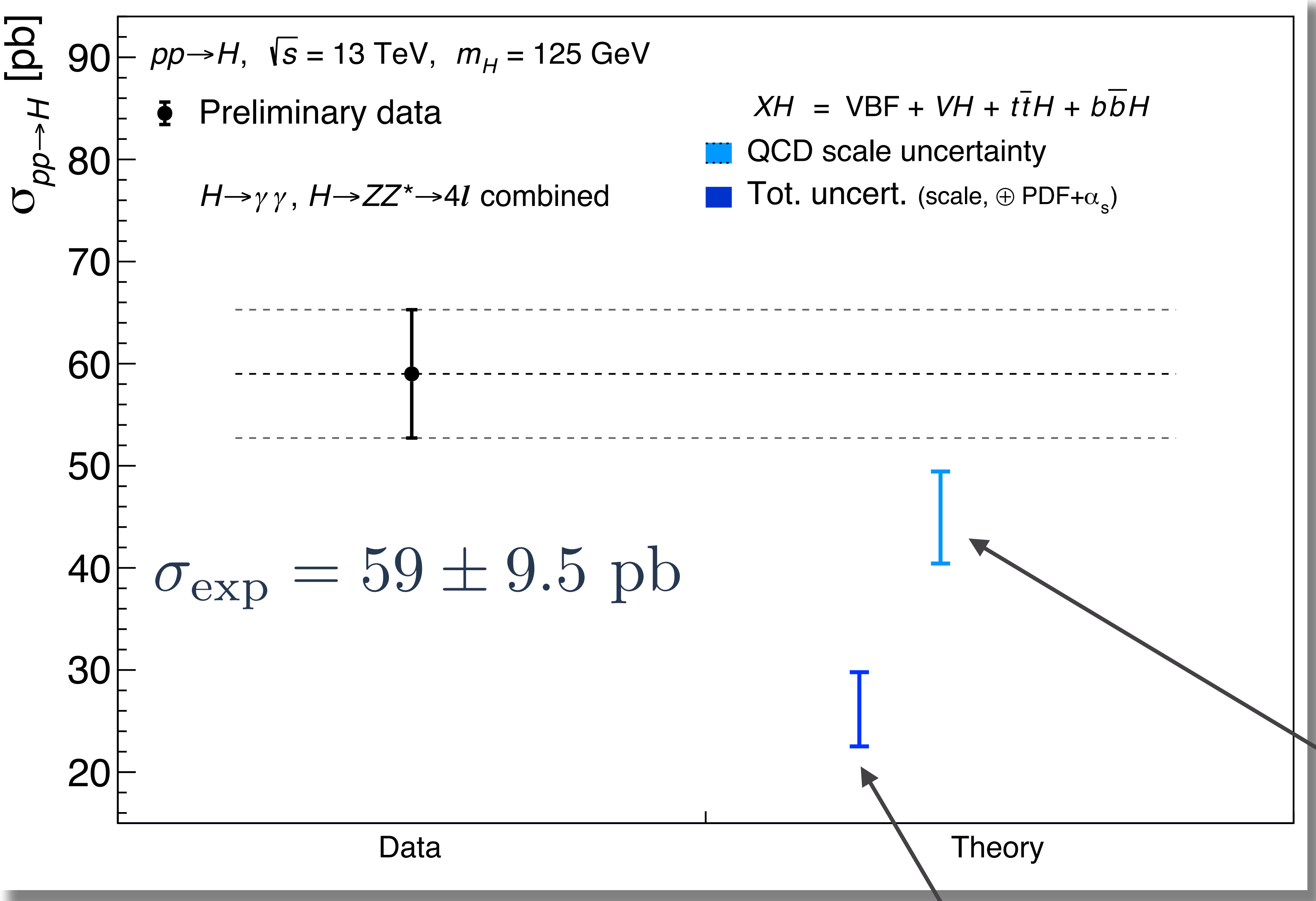
Higgs boson production: today $\sigma = \sigma_0$



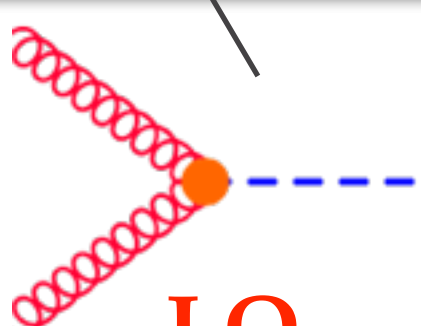
Precision and the Higgs

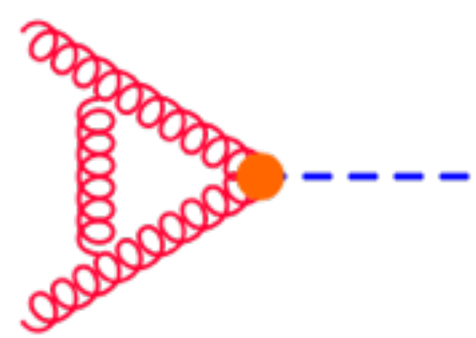
Higgs boson production: today

$$\sigma = \sigma_0 + \alpha_s \sigma_1$$



$$\sigma_{\mathrm{exp}} = 59 \pm 9.5 \text{ pb}$$

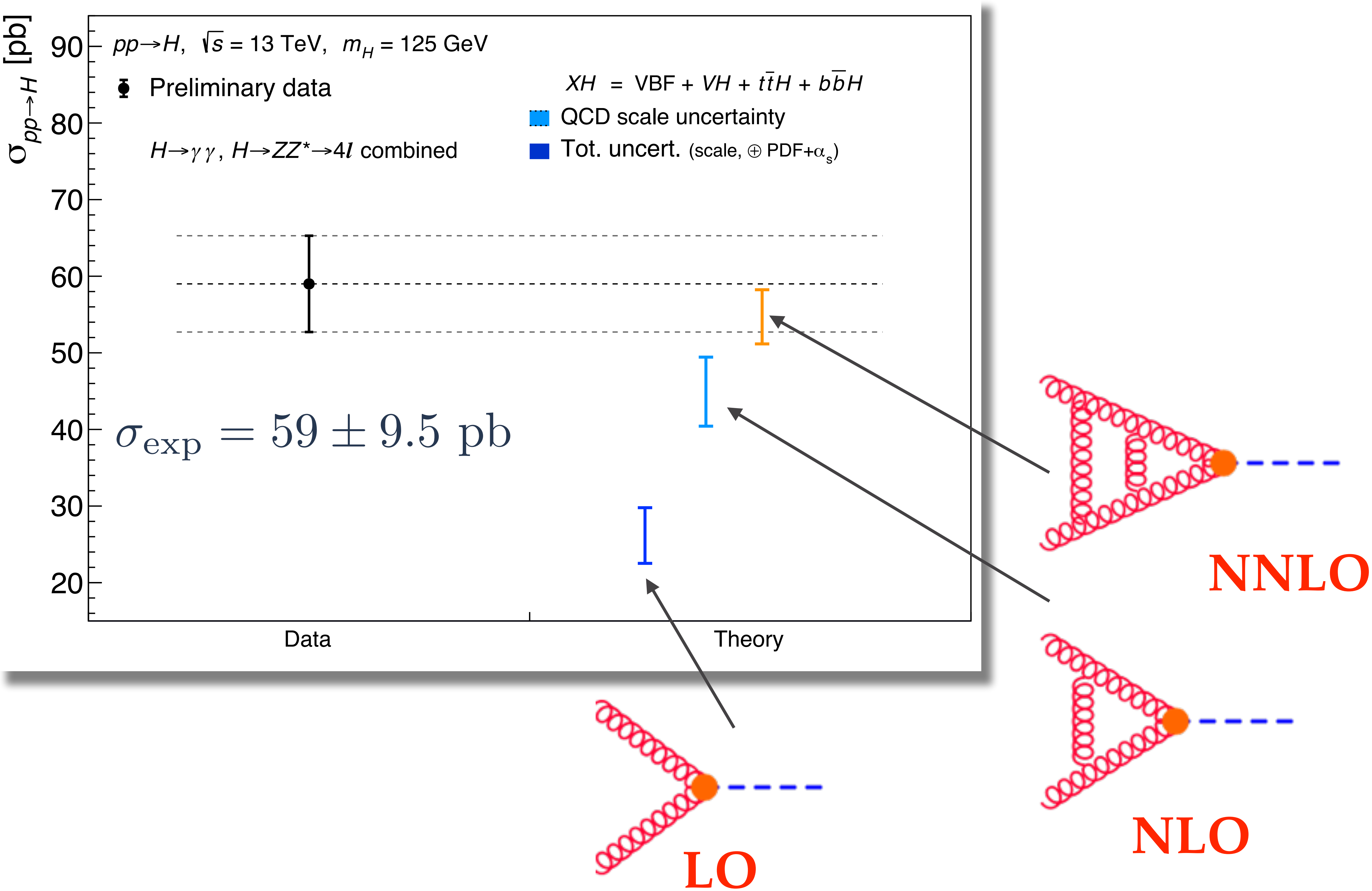




Precision and the Higgs

Higgs boson production: today

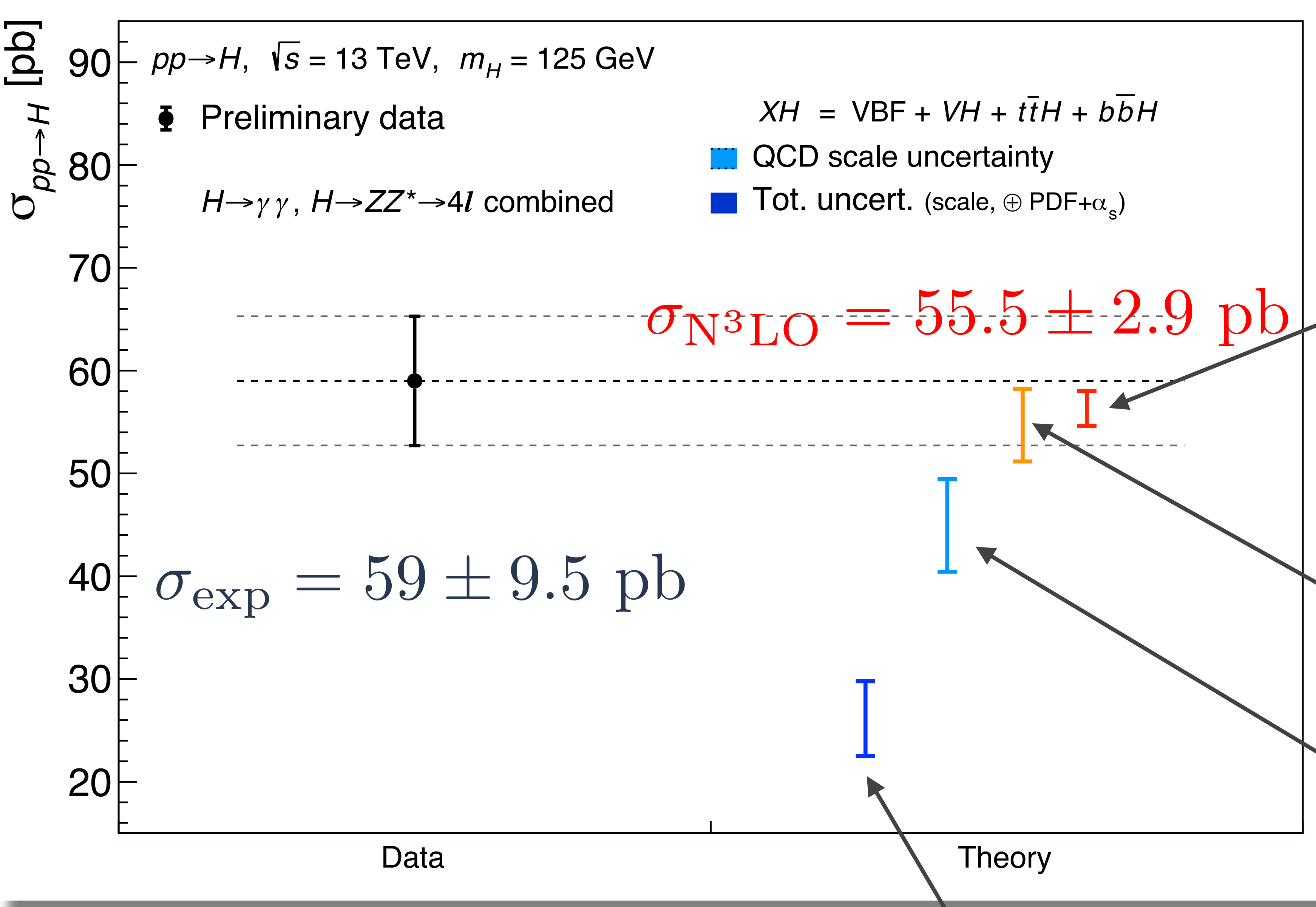
$$\sigma = \sigma_0 + \alpha_s\,\sigma_1 + \alpha_s^2\,\sigma_2$$

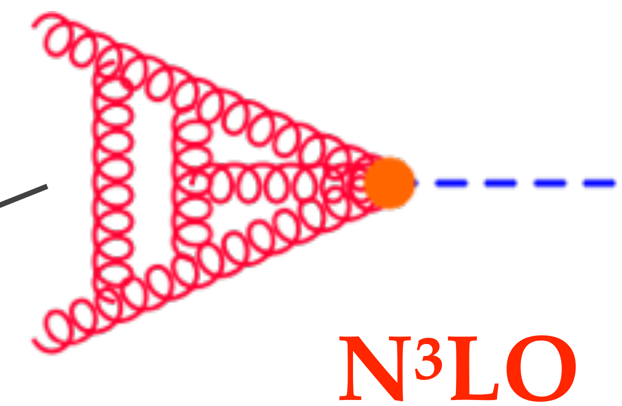


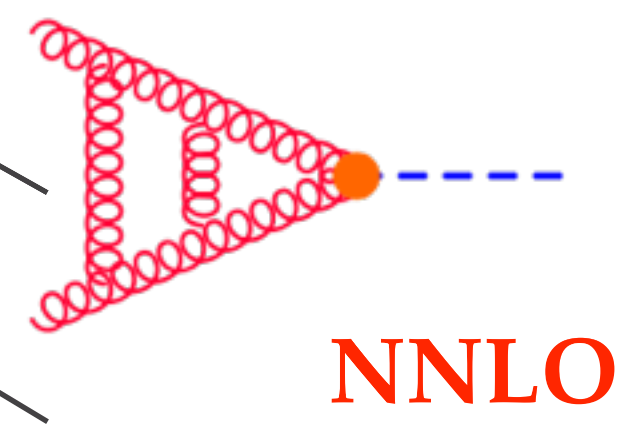
Precision and the Higgs

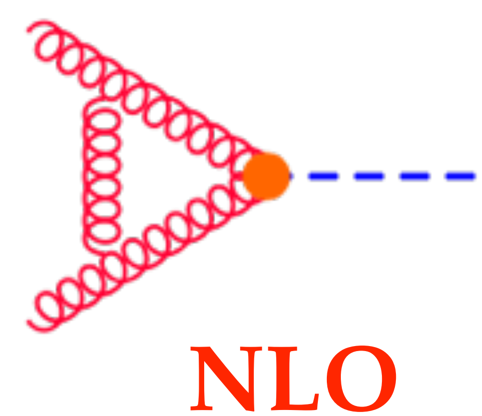
Higgs boson production: today

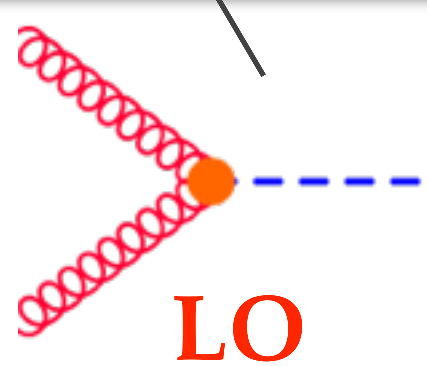
$$\sigma = \sigma_0 + \alpha_s\,\sigma_1 + \alpha_s^2\,\sigma_2 + \alpha_s^3\,\sigma_3$$







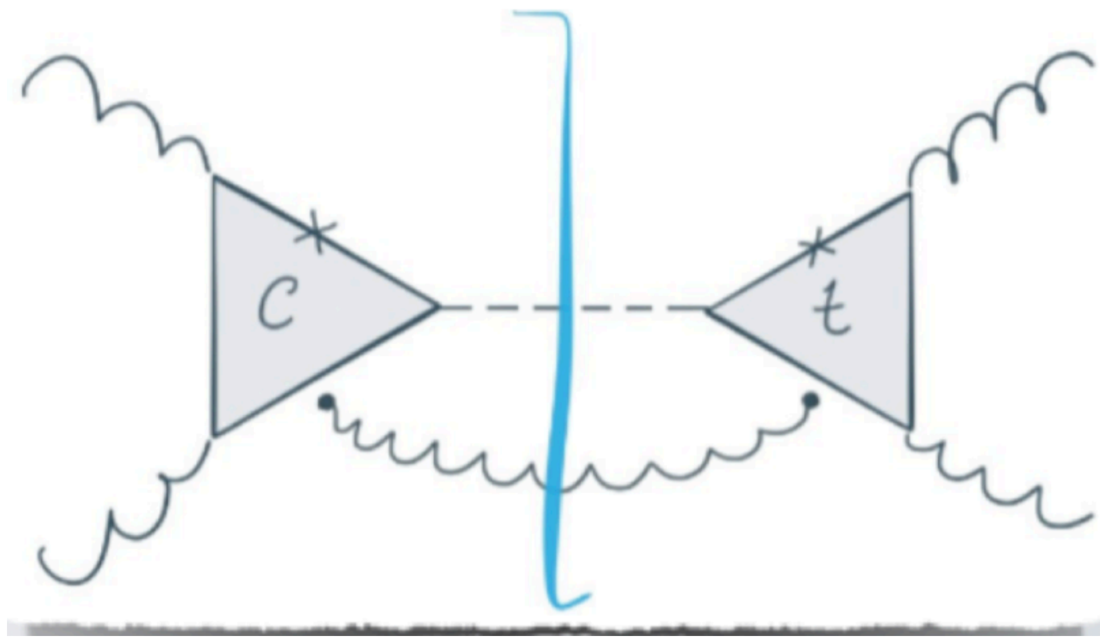


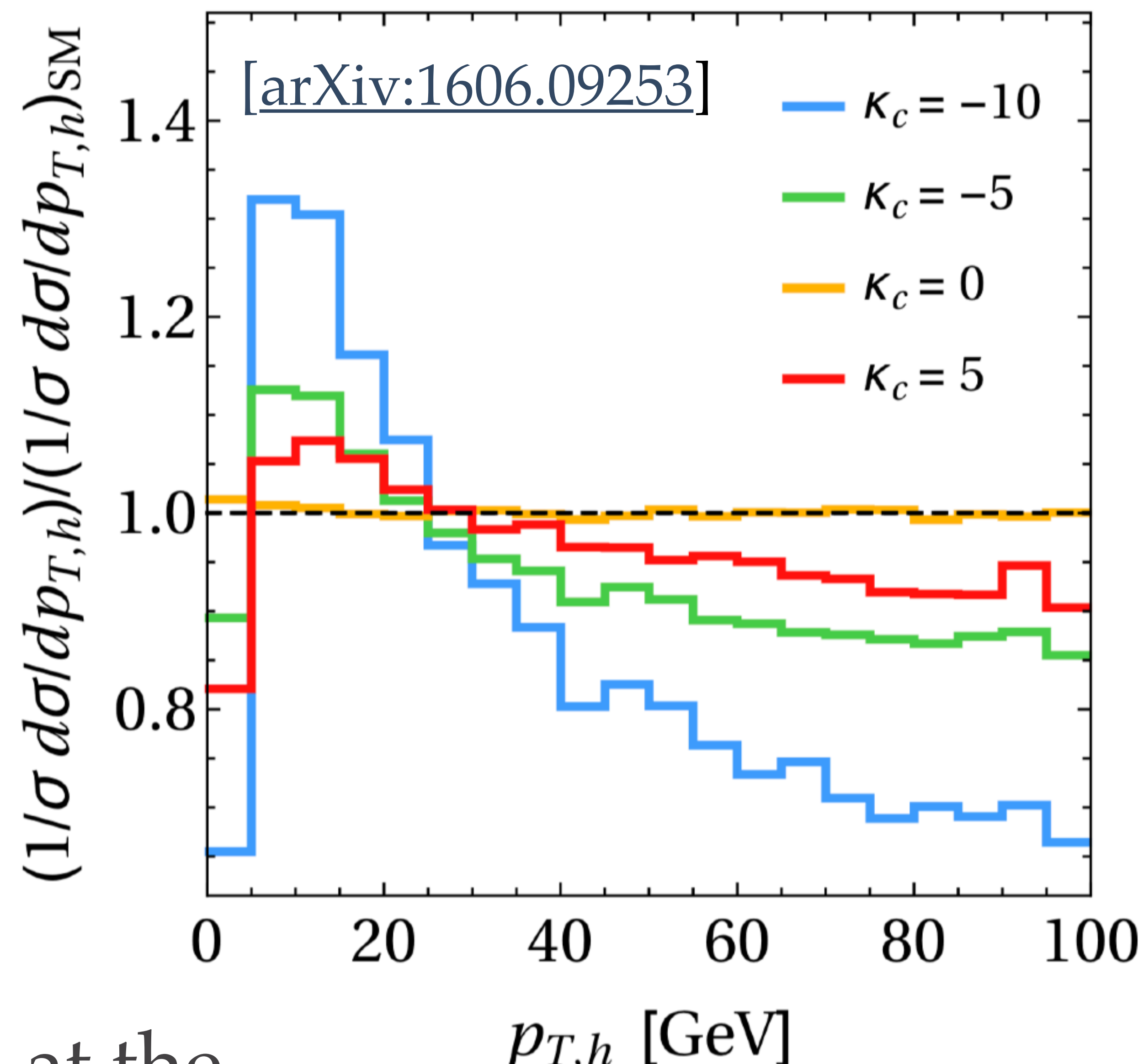


Very high orders required to test the Higgs sector!

Extracting Yukawa interactions

Use gluon as a probe to look inside Higgs interactions…

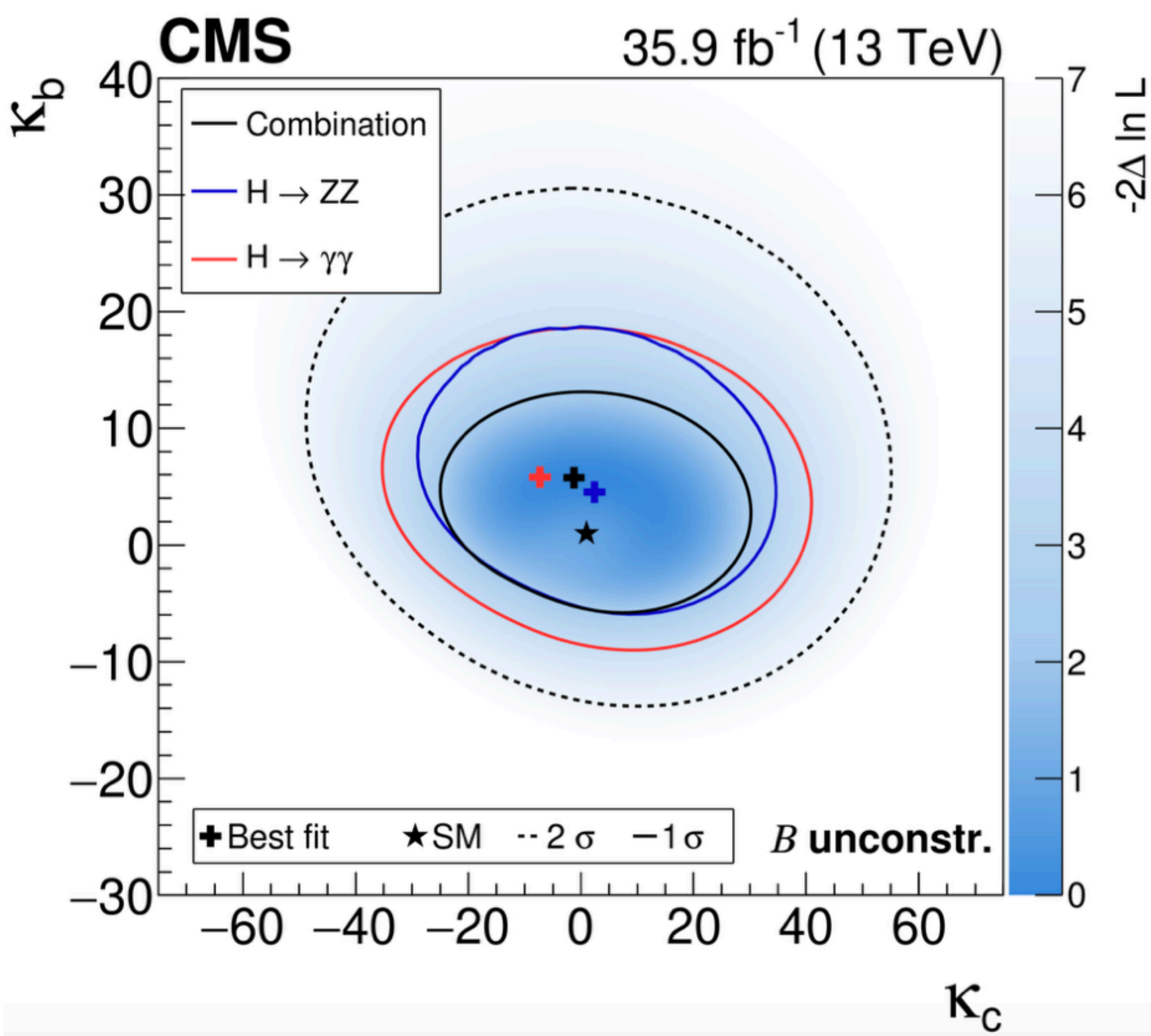


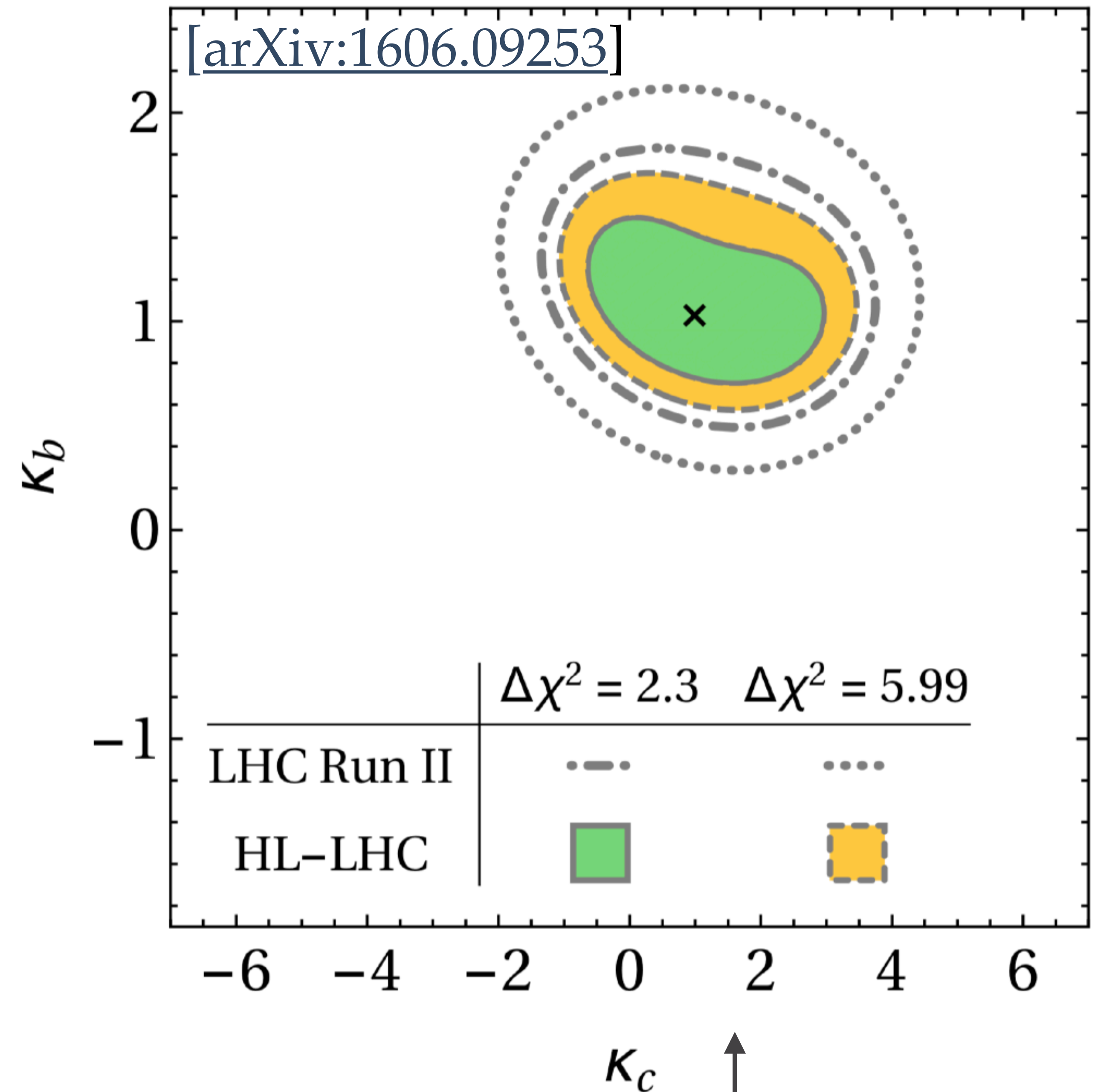


- ``1'': Standard Model
- Curves: Higgs is not as predicted by the Standard Model
- If we control theoretical predictions at the few percent → can disentangle

Extracting Yukawa interactions

Already interesting results





Could improve dramatically in the future

Conclusions

- Higgs studies at the LHC require very good control on QCD
- Although QCD is known since last century, a loot of exciting new progress
- Did cover only a tiny fraction of what is going on…
- A new way of looking at QCD, we keep learning interesting new stuff, and can apply it for high precision studies at the LHC

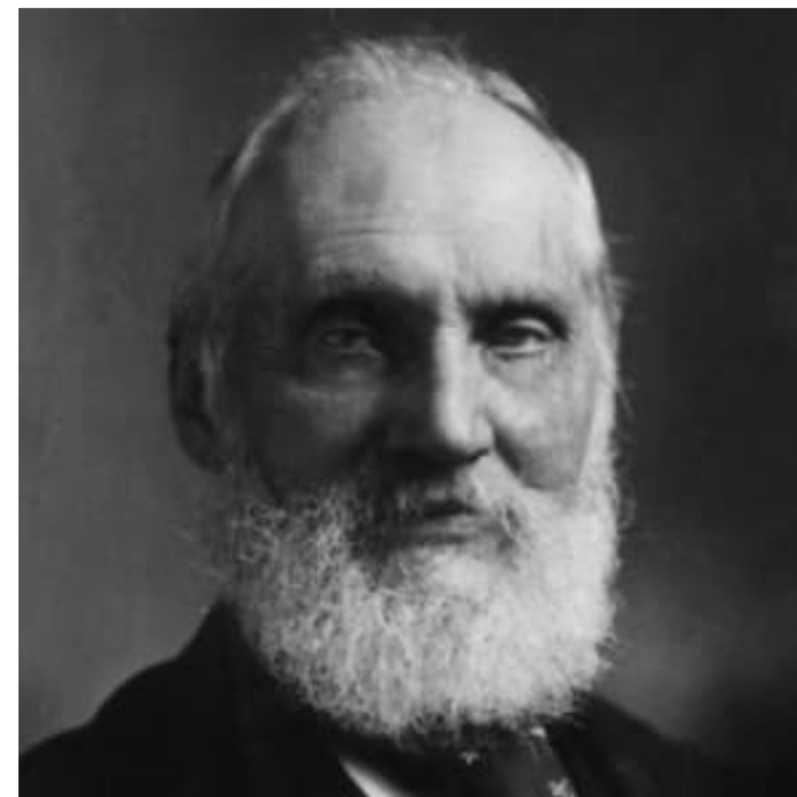
- Already know we can do a lot…
- … but in the precision program the best is yet to come: more data, better theory understanding
- LHC is the beginning of the Higgs story
- With precision, we can explore the very core of Higgs interactions

High precision Higgs studies

- With the Higgs, the Standard Model may be a complete theory. What is the point of looking at the next decimal digit?

> ``physics is complete, all we need to do is to measure some known quantities to a great degree of precision''

Lord Kelvin, ca 1900



5 years later: special relativity.

Less than 30 years later: quantum mechanics, general relativity

Thank you very much

… fatti non foste a viver come bruti, ma per seguir virtue e canoscenza

[*…you were not born to live like brutes, but to follow virtue and knowledge*]